

## 2015 - 16 EXPERIMENTAL PROJECT

### RESONANCE EXPERIMENT

#### 1. INTRODUCTION

Almost all mechanical systems can be seen to oscillate when tapped or hit. When a mechanical vibrator (a **driver** of frequency,  $f$ ) is applied to such a mechanical system, that system will, in general, vibrate with the **same** frequency,  $f$ , and with an amplitude,  $A$ , that is quite often small and has no significant consequences. However, some mechanical systems produce a greatly enhanced amplitude for a particular frequency, the **resonant frequency**, which is very close to the **natural frequency** of oscillation of the mechanical system. This phenomenon is known as **resonance**. Common examples are:

- (i) Shaking one end of a springy 30 cm plastic ruler in your hand. At low and high frequencies it does not do anything much, but if you shake it at its natural frequency, it will almost bend back and forth through  $90^\circ$ .
- (ii) A diver on a spring board jumps at the natural frequency of the board to maximize their momentum. i.e. the amplitude of their jump on the board keeps on increasing if they jump up and down at just the right frequency.
- (iii) An opera singer can produce a frequency that causes a wine glass to undergo resonance, causing it to shatter.
- (iv) A loose fixture in a car will vibrate, and rattle, when the car is travelling at a particular speed that produces the resonance vibration.

The experiments in this project investigate the **resonance** phenomenon for a simple pendulum.

#### 2. APPARATUS

- (i) A larger spherical mass,  $m_D = 0.5 - 3.0$  kg: the driving mass D, of metal or plastic.
- (ii) A smaller spherical mass,  $m_B = 5 - 30$  g: the bob mass B ( $m_D > 10 m_B$ ), of metal or plastic.

- (iii) A metre length of string to support the driver and bob pendulums at the top support end, Figure 1, and another metre of string to support the driver mass vertically.
- (iv) Cotton or thread from which the lighter bob mass hangs. Cotton can be attached using araldite or, for plastic spheres, a needle can be inserted into the plastic.
- (v) Two sturdy retort stands (with bosses and clamps), the tallest available, and two G-clamps (note the G-clamps are screwed from under the bench so as not to damage the paint on the retort stand).
- (vi) A point source of light to cast a shadow (e.g. filament bulb or spot lamp, or even a small desk lamp)
- (vii) Metre rule and a ruler, sheet of A4 paper and sellotape

### 3. EXPERIMENTAL ARRANGEMENT

Set up the two pendulums on the bench, as indicated in Figure 1. The bob, B, is suspended a vertical distance  $d$  below the string that supports the two pendulums, measured to the centre of B.  $d$  remains constant throughout the experiment, at a value that is set at roughly half the distance from the supporting string to the bench. B is supported using the cotton thread.  $D_1$ , the length of the driver pendulum, measured to the centre of D, will take a series of values; the smallest being no less than 5.0 cm. The pendulums should oscillate in planes perpendicular to the vertical plane containing the equilibrium directions of the strings (i.e. in and out of the paper in Figure 1).

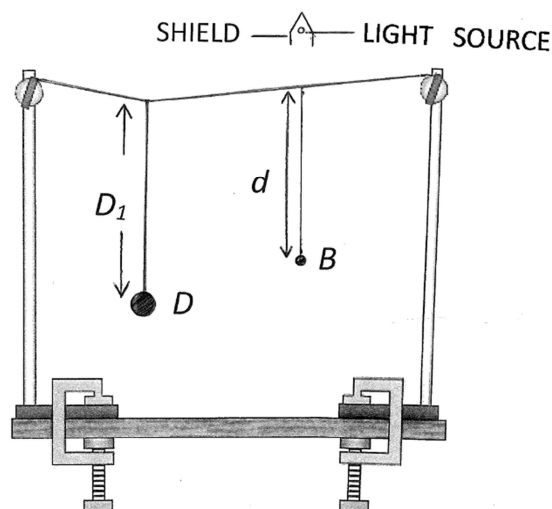


Figure 1

The point source of light should be positioned above the supporting string, and vertically above the bob, B, in order to cast a shadow of the bob onto the bench. It is helpful if it is shielded so that only the shadow of B can be easily viewed when in motion, and does not illuminate the whole room. Alternatively, the light source may be set up to the side of the retort stand, on the bench, in order to cast a shadow of the oscillating bob onto a vertical

screen or wall. The experiments should be performed in a *darkened* room in order to enhance the shadows. Shadow projection is one method of enabling measurements of pendulum amplitudes (from the centre of the swing to the maximum displacement from the centre or one half of the whole swing) and displacements to be conveniently made. Alternative methods may be devised in order to carry out the experiments if they are thought to be of comparable or greater accuracy.

In each experiment, D, with pendulum length  $D_1$ , is initially displaced horizontally a distance  $L$ , with its string taut, perpendicular to the vertical plane defined by the equilibrium directions of all the strings. It is held in this new equilibrium position. In doing so it has been raised vertically, from its initial equilibrium position, by a small **fixed** height,  $h$ , before being released to begin oscillating. This will ensure a constant initial energy for D, for each value of  $D_1$  cm, relative to its initial equilibrium position. One can substantially increase the accuracy of the determination of  $h$ , which is relatively small compared with  $L$ , by displacing D the appropriate horizontal distance,  $L$ , which is much greater than  $h$ . Initially B is stationary in equilibrium.  $h = 0.5$  cm for all experiments in this project. This requires one to set

$$L = [2hD_1 - h^2]^{\frac{1}{2}} \text{ cm} \quad (1)$$

#### 4. EXPERIMENT

The experiment is to be repeated for a series of values of  $D_1$ , starting from  $D_1$  a little greater than 5 cm, to a value where D is just above the bench. This may require from eight to twelve values of  $D_1$ , depending on the detail required and the shape of the graphs in Section 5. For each value of  $D_1$  determine:

- (i) the periods of oscillation of the driver, D, and the bob, B, following the release of D, once a steady dynamical state is reached. This will be after a few oscillations of D, when these periods are constant. Hence determine their frequencies,  $f_D$  and  $f_B$  respectively; this is best performed using the shadows of B and D.
- (ii) the horizontal maximum amplitude of oscillation of the bob, B, which is called  $A_1$ . This will occur *after a few initial oscillations* and can most conveniently be determined by measuring the displacement of the shadow amplitude,  $A_2$ , of the bob.

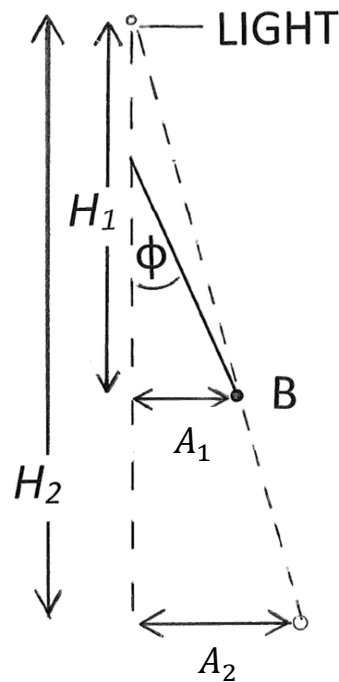
As shown in Figure 2, if  $H_1$  and  $H_2$  are the respective distances of the centre of the bob and its shadow measured from the point source of light, then, by similar triangles,

$$A_1 = A_2 \frac{H_1}{H_2} \quad (2)$$

Thus,  $A_1$  can be determined.

- (iii) Repeat (i) and (ii) with an open vertical paper cylinder wrapped symmetrically around the bob, secured with sellotape, which has a length equal to twice the diameter of the bob, in order to increase the effect of the viscous damping of the air. The axis of the cylinder should be along the direction of the bob string.

### DIAGRAM



**Figure 2**

Showing the illumination of B from above

### 5. RESULTS

Present tables with values of  $D_1$ ,  $f_D$ ,  $f_B$  and  $A_1$ , and plot graphs of:

- (i)  $A_1$  against  $f_B$  **without** the paper attachment to the bob.
- (ii)  $A_1$  against  $f_B$  **with** the paper attachment to the bob.

### 6. EXPERIMENTAL REPORT

This report should contain:

- (i) A concise description of the experimental procedure, including diagrams/photographs and the labels for your setup.

- (ii) A brief description of what you **observe** happening.
- (iii) An outline of any experimental techniques used to improve accuracy or reduce uncertainty, and/or modifications to the procedure based on any trial experiments.
- (iv) Precautions taken to consider safety.
- (v) Tables of measurements with headings and units.
- (vi) Errors and accuracy estimates and comments about them.
- (vii) A concluding discussion of results, units and accuracy.
- (viii) A photograph of the experimental arrangement.
- (ix) Calculations.

## 7. Report Guidelines

Some examples of what makes up some of the elements of a good write-up will be placed on the BPhO website this summer [www.BPhO.org.uk](http://www.BPhO.org.uk) . There is no single style, but a contribution of pieces of information which guide the reader through the process of experimentation. Keeping a log book, a decent size A4 notebook from which pages cannot be ripped out, and pasting in tables, graphs, comments and ideas, is important. It helps you keep track of your progress and keeps the information in one place. All professional experimenters (and theorists) keep such a notebook.

You should give a full but concise description of the experimental procedure. Highlight any particular ideas you had to make the results more reliable, with results, tables, units, uncertainties, graphs, diagrams and images. This is elaborated below.

You do not want a long report, but one which describes your progress and results in a way that would explain clearly to the reader what happens in the experiment, what you did, and what your results were, in such a clear and simple way that he is persuaded that you knew pretty much what you were doing. You are trying to remove elements of doubt in the reader's mind that you might just be making random measurements. Do you have a descriptive paragraph which describes how the two pendulums move over time, and how their swings relate to each other?

- You must include a plan of the experiment, such that a colleague could carry out the experiment from your instructions. You must also write about any additional points, the particular apparatus that you used, precautions you took and good ideas you had about reducing the uncertainties of your measurements and what were the difficult measurements to make and why. Do not write an essay but just a short comment about each good idea.

- Describe any trial measurements you made to see if the experiment was going to work, and what range of measurements you decided to take. Try out the apparatus to investigate what goes on and make sure that you observe how the pendulums behave; discuss it with someone to make sure that you see what there is to be seen.
- Say what changes or adjustments or range setting you made in the light of this experience.
- Take a **good range** of data, and **plot the graphs before you put the apparatus away**.
- Results tables should have the **original data** including any **repeated measurements**, with the **units** at the top of the column along with the **correct symbol** for the quantity. A column for the **average** can then be worked out. You might include a column with an **estimate of the uncertainty** on your average value.
- If you feel that the results are not right, then don't scribble them out or delete them; keep them and just point out what might be wrong with them or why you have no confidence in them. The purpose is to show that **you have investigated the apparatus** and given it a good try out so that you can convince the reader that **you know what you are talking about**. Things can go wrong and you need to show how you can overcome setbacks. If you delete everything that does not seem right to you, the reader might see a nice final table of results, but they do not know how much investigative effort you have put in to get that set of results. The reader might indeed wonder whether they are that good. Do **not** write an essay, just a few bullet point comments.
- Graphs should have labelled axes, units on the axes, with the plotted data taking up a significant proportion of the graph paper area, some gridlines if done in Excel so that the reader can get values off the graph, a scatter graph with a line of best fit (trendline in Excel) – definitely do not “join the points”. Display the equation for the trendline on the graph. You do not have to use Excel. You can draw graphs by hand, which are just as valid.
- **Comment on the graph** i.e. look at it and state what you can see about the shape of the graph, the scattering of the points, does it go anywhere near the origin, is it a straight line or a curve (or is it hard to tell), is the data close enough to the line of best fit to agree with the uncertainties you wrote down in your table of results?
- The **conclusion** comes from the table of results and your graph. What **trends** are shown, or what **general comments** can you make to **summarise** your results. You should also make at least a **comment** about whether you yourself think that the results are reliable or not, and why.

Good Luck with the experiment!