BRITISH PHYSICS OLYMPIAD 2014-15

A2 Challenge

September/October 2014

Instructions

Time: 1 hour.

Questions: Answer ALL questions.

Marks: Total of 50 marks.

Solutions: These questions are about problem solving. You must write down the questions in terms of symbols and equations, and try calculating quantities in order to work towards a solution.

In these questions you will need to explain your reasoning by showing your working. Even if you cannot complete the question, show how you have started your thinking; with ideas and, generally, by drawing a diagram.

Setting the paper: You are allowed any standard exam board data/formula sheet.

Important Constants

Speed of light	c	3.00×10^{8}	m s^{-1}
Planck constant	h	6.63×10^{-34}	J s
Electronic charge	e	1.60×10^{-19}	C
Mass of electron	m_e	9.11×10^{-31}	kg
Gravitational constant	G	6.67×10^{-11}	N m^2 kg^{-2}
Acceleration of free fall	g	9.81	m s^{-2}
Permittivity of a vacuum	ε_0	8.85×10^{-12}	F m^{-1}
Avogadro constant	N_A	6.02×10^{23}	mol^{-1}

Q1.

This question explores some consequences of a collision between bodies being perfectly elastic.

a)

i) A perfectly elastic collision conserves kinetic energy. What other physical quantity is also conserved in all collisions (perfectly elastic or otherwise)?

ii) A ball of mass, *m*, is dropped from rest at a height, *h*. State an expression for the kinetic energy of the ball at the end of this descent and hence derive an expression for its speed.

iii) Having fallen through a height, h, the ball makes a perfectly elastic collision with the floor. State an expression for its kinetic energy as it leaves the floor and hence deduce the height to which it rises before coming to rest. (ignore air resistance and similar forces)

You may observe that, for a perfectly elastic collision, the result

relative speed of approach = relative speed of separation

is implied in this situation (a special case of Newton's Law of Impact).

b) The experiment above is now repeated with two balls both of diameter negligible compared with the height, *h*, and with the smaller ball of negligible diameter compared with the larger one as in figure 1.1

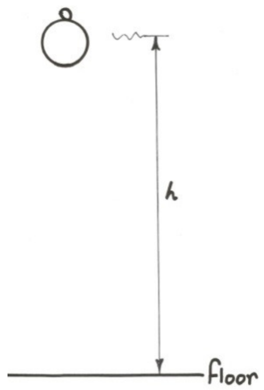

Figure 1.1

i) State the common *velocity* of the two balls as they reach the floor

ii) At the instant the *velocity* of the lower ball reverses direction, the upper ball is still falling. State the *velocity* of the lower ball and the relative speed of approach of the balls to each other at this moment.

iii) An instant later, the upper ball has rebounded from the lower ball without significantly reducing the speed of the much larger, lower ball. State the relative speed of separation of the two balls and hence deduce the upward speed of the smaller ball immediately after the collision.

iv) Hence deduce the final height to which the smaller ball rises.

c) A child's toy consists of a suitably mounted sequence of balls of decreasing size, made from a material which to all intents and purposes undergoes perfectly elastic collisions, as in figure 1.2.

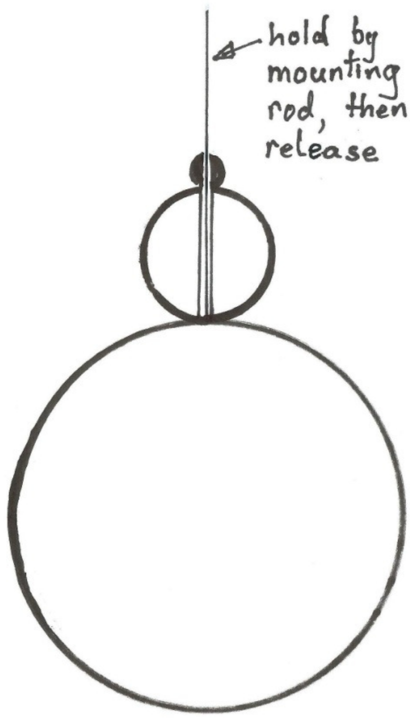

Figure 1.2

The toy is dropped from a height of 1m. How many balls would the toy need to contain for the top one to exceed escape velocity, which is approximately 11 kms^{-1} ?

14 marks

Q2.

This question refers to a novel version of the two-source interference pattern, commonly called *Young's Fringes*.

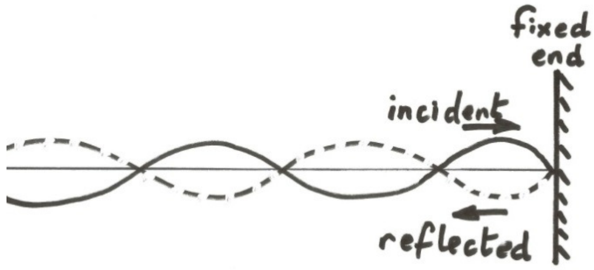

Figure 2.1

a) Figure 2.1 shows a transverse wave on a string being reflected from a fixed barrier.

   i) State the resultant displacement of the string immediately adjacent to the reflecting surface.

   ii) The incident and reflected waves together form a standing (or stationary) wave. What feature of a standing wave is situated at the reflector?

   iii) What is the phase difference beween the incident and reflected waves at distances of 0, $\lambda/4$ and $\lambda/2$ from the reflector? (Note: this result only applies to transverse waves)

b) Figure 2.2 shows a variation of the usual *Young's slits* apparatus called *Lloyd's mirror*.

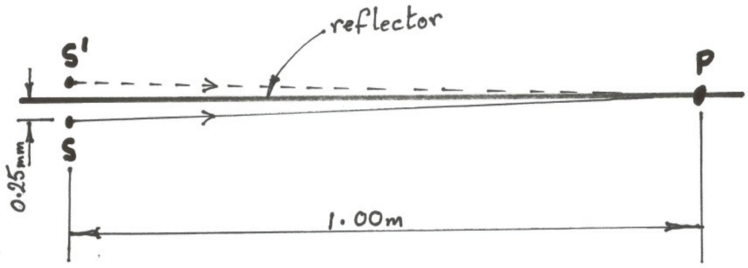

Figure 2.2

In this arrangement, S is a single small light source 0.25mm from the reflector. S', which constitutes the second source, is the image of S in the reflector.

i) Is S' real or virtual?

ii) Explain how this arrangement ensures that S, S' are coherent, as required to form the *Young's fringes* interference pattern in the space below the mirror.

iii) Why will there not be an interference pattern in the space above the mirror?

iv) At P, 1.00m from S,S', an interference pattern is observed with a fringe spacing of 1.00mm. What wavelength of monochromatic light was used to form this fringe pattern?

v) P is a point on the mirror and so must give the geometrical path difference, *(SP-S'P)* as zero. This is the position of the so-called central fringe. Bearing in mind your answer to (a-iii) above, comment on the intensity of this fringe.

vi) Comment on how this compares with the intensity of the central fringe formed using the conventional double-slit arrangement.

13 marks

Q3.

This question relates to networks of resistors.

a)

i) State the formula for the equivalent resistance of two resistors, R_1 and R_2 connected in parallel.

ii) Show that this leads to the '*product over sum*' rule:

$$Equivalent\ resistance = R_1R_2/(R_1 + R_2)$$

(You may find this is a convenient short-cut later in the question)

b) Figure 3.1 shows a unit used to build a system known as a *ladder network*.

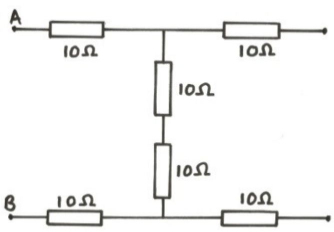

Figure 3.1

i) Calculate the resistance between points A and B

ii) A second unit is added on the left as in figure 3.2

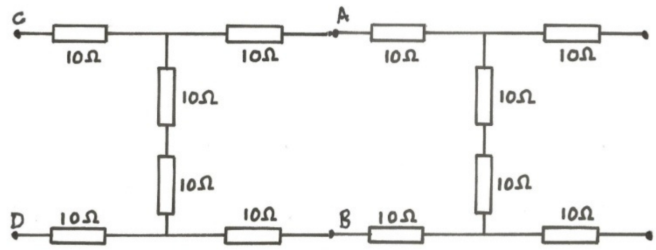

Figure 3.2

Now calculate the resistance between C and D.

iii) It would be possible to add a *very* large number of such units to form a *ladder network* of resistance *r*, as in figure 3.3. By this stage, adding one extra unit would make no significant difference to the resistance between Y and Z.

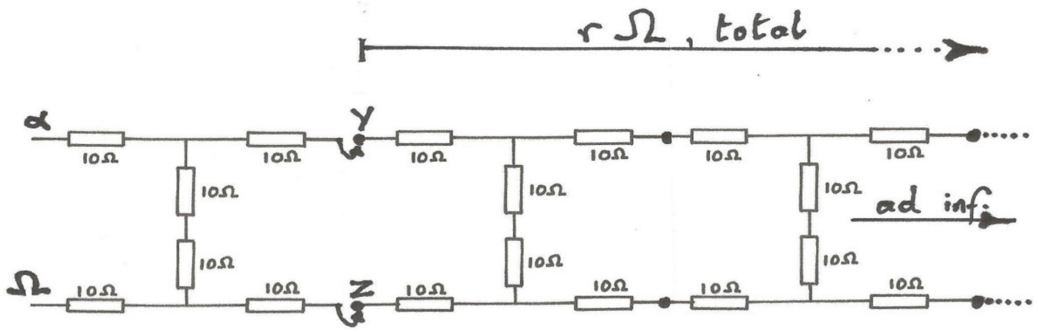

Figure 3.3

Now, calculate the resistance between **α** and **Ω** when one extra ladder unit is added to the left of Y,Z as suggested by the small arrows in figure 3.3.

iv) Use your result to show that the equivalent resistance of an infinite ladder made of these units is $20\sqrt{3}\,\Omega$.

9 marks

Q4.

This question examines two examples of bodies heated uniformly throughout their volume.

a)

i) What can you say about the temperature of the surface of a body which loses thermal energy to its surroundings?

ii) The Earth is heated throughout its volume by natural radioactivity, then radiates this energy into space. Develop your ideas from (a-i) above to comment on the variation of temperature as depth below the Earth's surface increases.

iii) These radioactive processes liberate thermal energy at a rate ρ per unit volume. State the total rate of production of thermal energy inside a solid sphere of radius, *r*, concentric with and contained within the Earth.

iv) By considering the thermal energy flowing out through a thin spherical shell of radius, *r*, and thickness, *δr,* find the temperature difference, *δT,* across the thin shell.
(You will need the relationship:

$$\text{Rate of thermal energy transfer through a thin layer of material (thickness } \delta\text{r)} = -k\frac{\delta T}{\delta r} \times surface\ area\ of\ layer$$

where *k* is a constant, called the thermal conductivity of the material through which thermal energy is being transferred.)

v) Hence determine the relationship between distance from the Earth's centre, *r*, and temperature, *T*. (This requires the integration of the relationship you have derived for *δT/ δr* in part (iv) of this question)

vi) Sketch the temperature profile of the Earth between the centre and the surface.

b) Damp haystacks also generate thermal energy throughout their volume due to fermentation processes. Comment on why this may lead to spontaneous combustion and why it is inadvisable to break open an overheating haystack in order to cool it down.

14 marks

End of Questions