

BPhO

British Physics Olympiad

British Physics Olympiad 2018-19

A2 Challenge

September/October 2018

Instructions

Time: 1 hour.

Questions: Answer ALL questions

Marks: Total of **50 marks**.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used.

Solutions: These questions are about problem solving. Draw diagrams in order to understand the questions. You must write down the questions in terms of symbols and equations; then try calculating quantities in order to work quickly towards a solution. In these questions you will need to explain your reasoning by showing clear working. Even if you cannot complete the question, show how you have started your thinking, with ideas and, generally, by drawing a diagram.

Clarity: Solutions must be written legibly and set out properly with a “narrative” which links one step to the next (and, so, therefore, hence, but, also, using equ 5, etc.).

Important Constants

Constant	Symbol	Value
Speed of light in free space	c	$3.00 \times 10^8 \text{ m s}^{-1}$
Elementary charge	e	$1.60 \times 10^{-19} \text{ C}$
Acceleration of free fall at Earth's surface	g	9.81 m s^{-2}
Planck's constant	h	$6.63 \times 10^{-34} \text{ J s}$
Avogadro's constant	N_A	$6.02 \times 10^{23} \text{ mol}^{-1}$
Density of water	ρ_w	1000 kg m^{-3}

Qu 1. This question is about a practical method for finding the e.m.f. and internal resistance of a power source.

Figure 1 shows a source of emf, E , and internal resistance, r , connected to a calibrated variable resistor of resistance, R . A set of readings is taken of the current, I , and resistance, R , as the value of R is varied.

- (i) Compose an equation relating E , I , r and R .
- (ii) Re-arrange your equation into a linear form which includes the two variables I and R , and thus state which variables you would plot to find E and r . State how you would deduce the required results from your graph.
- (iii) Show that the maximum rate of heat dissipation in R occurs when $R = r$. (It may help you to write down the equation relating the potential V across the variable resistor and the current I).

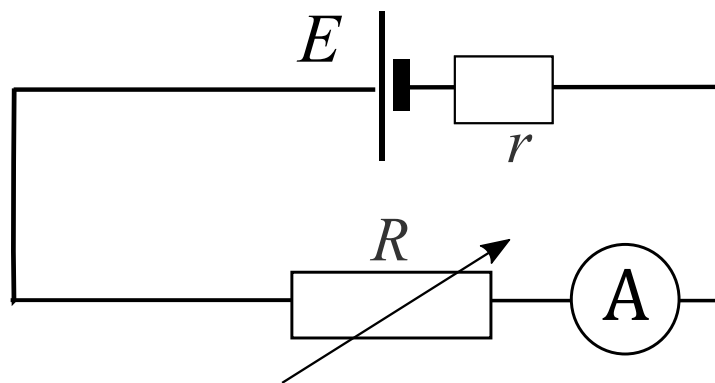


Figure 1

[6]

[6 marks]

Qu 2. This question examines some ideas and misconceptions about kinematics.

- a) A physics student describes a journey, of total length 60 km, to a friend who dislikes mathematics. He travels the outward journey at 60 km h^{-1} and the return journey at 30 km h^{-1} .
- (i) What average speed for the round-trip will the physics student calculate?
 - (ii) The friend objects, by saying that the average of 60 km h^{-1} and 30 km h^{-1} is 45 km h^{-1} , rather than the answer the physics student has correctly reported. Explain why the physics student is right and his friend is mistaken.

[2]

- b) The friend has now decided to take up target shooting as a hobby and has purchased a brand new rifle. A bullet has a velocity of 400 m s^{-1} as it leaves this type of rifle. Upon arriving at his shooting club, he aims very carefully at a target 100 m away and at the same horizontal level as himself, making sure that the barrel of the gun is perfectly horizontal and facing directly at the target. There is no wind and the effects of air resistance may be neglected.
- (i) Explain why he misses the desired target point every time.
 - (ii) How far away, and in what direction from the desired point of impact does his shot fall?

After travelling 100 m the bullet only just misses the target. The physics student tells his friend that he needs to aim at a slightly different angle.

- (iii) Sketch the path of the bullet if it is fired at a suitable angle to hit the target.
- (iv) Exactly halfway on its path to the target we want the vertical speed of the bullet to be zero. What would be the initial vertical speed of the bullet as it leaves the rifle? (You may assume that the horizontal speed is unchanged).
- (v) Knowing the speed of the bullet at the start, and your answer from part (iv), determine the direction in which the rifle should be fired in order for the bullet to reach the desired target point.

Much heartened by doing this, the friend decides to try the same technique on a more advanced target exercise where the target, still 100 m away, is allowed to fall freely at exactly the instant the rifle fires. To his annoyance, he again misses every time.

- (vi) In what direction he should now aim to hit the falling target? And why?

[6]

- c) Being an historian, the friend, who is keen on battle re-enactments, and cannot be bothered further with the technicalities of adjusting his rifle-sights, decides to try his hand with an old-fashioned musket, having a muzzle velocity of only 30 m s^{-1} . He now finds that wherever he aims he cannot hit the target.
- (i) Derive an expression for the horizontal range of a projectile launched at a velocity, v at an inclination of θ to the horizontal.
Hint: You may use the formula $2 \sin \theta \cos \theta = \sin 2\theta$
 - (ii) Hence deduce how the physics student would explain the cause of his friend's present difficulties.
 - (iii) Now use your expression to calculate the angle at which the 400 m s^{-1} bullet from the first rifle must be fired to hit the target 100 m away.

[5]

[13 marks]

Qu 3. This question relates to the familiar kinetic theory for ideal gases and looks into some circumstances in which it does not completely reflect reality.

- a) We will first examine a much simplified version of the kinetic theory of gases, in which N molecules in a cubic box of side a , make elastic collisions with their container walls. For simplicity, it is supposed that $\frac{1}{6}N$ molecules, each of mass m , are travelling at speed c , directly towards each wall, as in **Figure 2**.

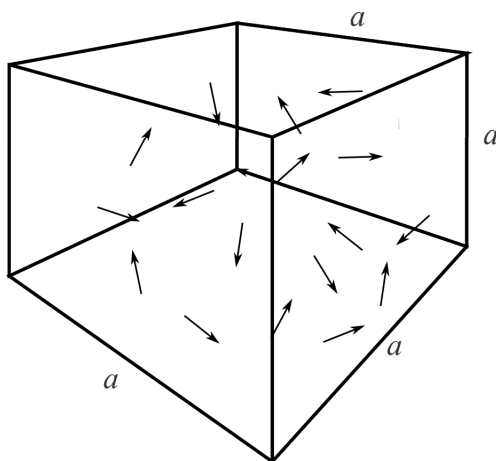


Figure 2

- (i) What is total momentum of all the molecules that are travelling vertically upwards?
- (ii) What is the momentum change when all these molecules have rebounded elastically off the top of the box once?
- (iii) How much time will elapse before all the molecules currently moving upwards have struck the top of the box? (Assume they are uniformly distributed throughout the box)
- (iv) Hence find the average force on the top of the box.
- (v) Therefore, what is the pressure p on the top of the box?

In fact, the shape of the box is immaterial, so we could simply write V for its volume. Now, the relationship between absolute temperature T and molecular kinetic energy is defined by

$$\text{average molecular kinetic energy} = \frac{3RT}{2N_A}$$

where R is the molar gas constant and N_A is the Avogadro constant.

- (vi) Use this information to find a relationship between the variables p , V , N and T .
- (vii) Hence derive the familiar form of the universal (combined) gas law, $pV = nRT$ in which n is the number of moles of gas.

[8]

- b) One assumption contained in the calculation above is that the molecules are point masses. We will now examine how true this is. One mole of water has a mass of 18 g. One mole of any gas at room temperature and standard atmospheric pressure occupies approximately 24 dm^3 . ($1 \text{ dm} = 10 \text{ cm}$).

- (i) When supersaturated water vapour at room temperature is condensed, what fraction of its original volume does it occupy?
- (ii) Hence deduce the ratio of the linear size of the molecules to their average separation in a gas, explaining any assumptions you make.
Clearly, the space available to molecules is less than V by a small but significant proportion.
- (iii) What would the answer to b(ii) become if the gas were compressed to a pressure of 100 times normal atmospheric pressure? Again indicate any assumptions you make.

- (iv) At approximately what pressure would you expect a highly compressed gas to have structure indistinguishable from the corresponding liquid?

To allow for the reduction in the volume of space in which the molecules may move, we might write the ideal gas equation thus:

$$p(V - b) = nRT$$

One possible approach to estimating the term b is to imagine the gas molecules as spheres of radius r and to assume that they might only collide with each other in pairs i.e. collisions of three and more molecules simultaneously have negligible probability. Two such colliding molecules are shown in **Figure 3**.

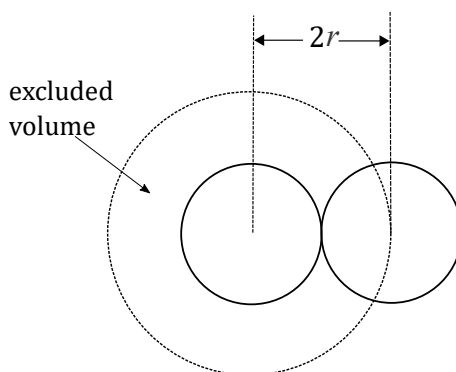


Figure 3

- (v) State the volume from which the centre of the right-hand molecule is excluded by the presence of the left-hand molecule.
 (vi) Hence state the excluded volume per molecule.
 (vii) Therefore what is b in terms of n , N_A and r ?

[10]

- c) The other way in which the ideal gas model differs from reality is in the lack of intermolecular attractions.

- (i) What effect will mutual attraction between molecules have as a gas is progressively cooled?
 (ii) At any time, such attractions will have an effect, Δp , on the pressure, p , which the gas is observed to exert on the walls of its container. Re-write the equation from part b(iv) above, incorporating the term Δp appropriately.

One approach to estimating Δp is to say that it is related both to the frequency of molecular impacts (as each impact is reduced by the attractions). Also Δp is related to the concentration of molecules (the number per unit volume) as closer proximity will increase the attractive forces.

- (iii) Suggest proportionalities which each the above two factors have with the volume, V , of the gas.
 (iv) Hence suggest a proportional relationship between Δp and V using the symbol a as the constant of proportionality.
 (v) Combine your answers to (ii) and (iv) above to produce an equation of state for a real gas. (This result is called van der Waals equation, published in 1873.)

[6]

[24 marks]

Qu 4. This question analyses an ingenious method of avoiding the systematic errors inherent in simple methods of determining the specific heat capacity of a liquid, using what is termed a *constant flow method*.

In **Figure 4** is shown the apparatus used for Callendar and Barnes' method in which water flows steadily through the central pipe as shown, heated by the electric element, marked as the heater. The platinum resistance thermometers are sensitive electrical thermometers to measure the temperature of the water flowing in and flowing out. An electrical circuit with a battery, ammeter and voltmeter is connected to the heating element.

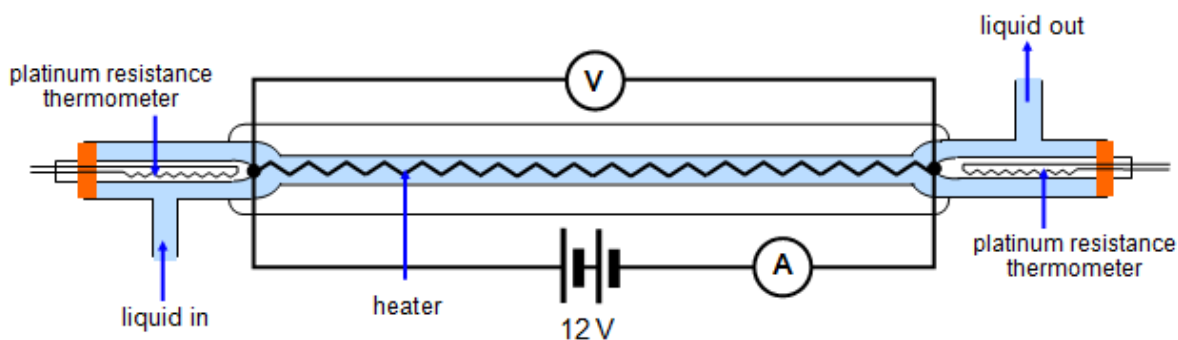


Figure 4: Callendar and Barnes' apparatus. Credit: <http://www.schoolphysics.co.uk>

In use, the apparatus is allowed to run for a while until a steady state is reached before the first measurements are taken. The flow rate is then changed and the power input adjusted until the temperatures are restored to exactly the same values as previously. A second set of measurements is then made.

- (i) State the power input to the heater in terms of the potential difference, V and the current, I as measured on the two meters.
- (ii) The rate of energy absorption by the water P is given by the expression:

$$P = mc\Delta\theta$$

where m is the **mass flow rate**, c is the specific heat capacity of the water and $\Delta\theta$ is the increase of temperature, as recorded by the two thermometers. Suggest why this is always less than your answer to (i) above and explain why the energy loss rate, L , is always the same when the first and second sets of observations are made.

- (iii) Compose an equation relating V , I , L , m , c and $\Delta\theta$.
- (iv) Now write two versions of this equation relating the first and second sets of observations, by suffices such as V_1 , V_2 , etc.
- (v) Subtract your two equations and gather together any like terms. Show that this method removes the systematic error represented by L .

[7]

[7 marks]

END OF PAPER

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