BPhO

British Physics Olympiad

BPhO Round 1

Section 2

13th November 2020

This question paper must not be taken out of the exam room

Instructions

Time: **5 minutes reading time (NO writing)** and then **1 hour 20 minutes** for writing.

Questions: Only **two questions** out of the five questions in *Section 2* should be attempted.

Each question contains independent parts so that later parts should be attempted even if earlier parts are incomplete.

Working: Working, calculations, explanations of the physics and **diagrams**, properly laid out, must be shown for full credit. The final answer alone is not sufficient. Writing must be brief but clear. If derivations are required, they must be mathematically supported, with any approximations stated and justified.

Marks: Students are recommended to spend about 40 minutes on each question. Each question in *Section 2* is out of 25, with a **maximum of 50 marks from two questions** only.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

Solutions: Answers and calculations are to be written on loose paper **ON ONE SIDE ONLY**. These will be scanned.Students should ensure that their **name** and their **school/college** are clearly written on each and every answer sheet. Number the pages.

Setting the paper: There are two options for sitting BPhO Round 1:

a. *Section 1* and *Section 2* may be sat in one session of 2 hours 40 minutes plus 5 minutes reading time (for *Section 2* only). *Section 1* should be collected in after 1 hour 20 minutes and then *Section 2* given out.

b. *Section 1* and *Section 2* may be sat in two sessions on separate occasions, with 1 hour 20 minutes plus 5 minutes reading time allocated for *Section 2*. If the paper is taken in two sessions on separate occasions, *Section 1* must be collected in after the first session and *Section 2* handed out at the beginning of the second session.

Important Constants

Constant	Symbol	Value
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Mass of electron	m_{e}	$9.110 \times 10^{-31}\,\mathrm{kg}$
Mass of proton	m_{p}	$1.673 \times 10^{-27}\,\mathrm{kg}$
Mass of neutron	m_{p}	$1.675 \times 10^{-27}\,\mathrm{kg}$
atomic mass unit	u	$1.661 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV\,c^{-2}}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}}$
Earth's gravitational field strength	g	$9.81\,\mathrm{N\,kg^{-1}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Avogadro constant	N_{A}	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Gas constant	R	$8.3145\,\mathrm{J\,K\,mol^{-1}}$
Mass of Sun	M_{S}	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Earth	R_{E}	$6.37 \times 10^{6}\,\mathrm{m}$

$$T_{(\mathrm{K})} = T_{(^\circ\mathrm{C})} + 273$$

$$\text{Volume of a sphere} = \tfrac{4}{3}\pi r^3$$

$$e^x \approx 1 + x + \ldots \qquad x \ll 1$$

$$(1+x)^n \approx 1 + nx \qquad x \ll 1$$

$$\frac{1}{(1+x)^n} \approx 1 - nx \qquad x \ll 1$$

$$\tan\theta \approx \sin\theta \approx \theta \qquad \text{for } \theta \ll 1$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2} \qquad \text{for } \theta \ll 1$$

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Section 2 — attempt two questions only

Question 2

This question is on the photoelectric effect and radioactive decay.

a) When UV light is shone on a metal surface electrons are emitted.

(i) Write down an equation which illustrates conservation of energy for this situation, relating the work function W, to the kinetic energy of the emitted electrons and the wavelength, λ, of the monochromatic light incident on the metal.

Two plane, large area, parallel conducting plates are placed $2.0\,\mathrm{cm}$ apart in a vacuum. The plates form part of a circuit containing a very sensitive ammeter (and amplifier) such that a current as low as $0.1\,\mathrm{pA}$ can be measured. One of the metal plates is illuminated with UV light of wavelength $280\,\mathrm{nm}$, causing photoelectrons to be emitted. The current flow is reduced to zero by an applied potential of $-0.82\,\mathrm{V}$ relative to the emitting plate.

(ii) How many electrons per second can the ammeter detect?

(iii) Calculate the work function of the metal plate.

(iv) Determine the speed of the most energetic photoelectrons emitted from the plate.

(v) If one of the photoelectrons with the maximum kinetic energy is emitted at an angle of $20°$ to the normal to the surface, calculate how far from its starting point it returns to the plate.

(5)

b) A $300\,\mathrm{m}$ long evacuated pipe has a parallel beam of very low intensity monochromatic light from a laser of wavelength $585\,\mathrm{nm}$ directed along its axis from one end. A laser has a power of $1.2\,\mathrm{mW}$ and the power reaching the far end is required to be less than $6.6 \times 10^{-11}\,\mathrm{W}$.
This can be achieved by a set of absorbing filters, each filter reducing the light intensity by a factor of 4.5.

(i) Calculate the minimum number of filters required to reduce the power to that required value.

(ii) State the resulting power of the light obtained.

(iii) Calculate the average number of photons reaching the far end each second.

(iv) Determine the average distance between photons travelling down the pipe.

(v) If a Young's double slit arrangement was placed at the end of the tube and illuminated with this low intensity light, and the intensity pattern was recorded on a sensitive camera placed behind the slits, what would be the appearance of the pattern recorded on the camera?

(vi) How would your answer to part (iv) differ if the pipe was filled with a transparent medium of refractive index $n = 1.2$?

(8)

c) An uncharged copper sphere of radius $8.0\,\mathrm{cm}$ hangs from a thin, insulating thread in a vacuum. It is illuminated with light covering a range of wavelengths from $600\,\mathrm{nm}$ down to $150\,\mathrm{nm}$ and becomes charged.
The work function of copper is $4.7\,\mathrm{eV}$.

(i) What is the value of the final potential of the sphere?

(ii) Calculate the number of electrons that have been lost from the sphere.

(iii) If the light illuminating the sphere is reduced to a very low intensity, it is calculated that an electron might be emitted every 0.1 s on average. However, measurements show that an electron may be emitted a few nanoseconds after the illumination is switched on. Explain this observation.

(iv) The illumination is switched off and it is observed that the sphere discharges by a small current flowing through the supporting thread, which is not a perfect insulator. What is the value of its resistance if the sphere loses half of its charge in $20\,\mathrm{s}$?

(4)

d) The half-life of the long lived isotope Radium-226 was originally measured by allowing it to charge an insulated metal plate and measuring the time taken to reach a known potential, repeatedly, over a long period of time. Such an apparatus is shown schematically to the right in **Figure 1**. The apparatus is supported on insulating rods and contained in a vacuum.

The Ra-226 is contained in the thin walled glass container in the form of $3.62\,\mathrm{mg}$ of newly prepared radium bromide (RaBr). As it beta decays, the metal plate that it sits on charges up and the flexible golf leaf rises until it reaches $220\,\mathrm{V}$, when it touches the earthed electrode and is discharged. Assume that all beta particles escape through the glass and away from the apparatus. Measurements on the apparatus show that the potential on the gold leaf is proportional to the charge, with the constant of proportionality being $6.40\,\mathrm{pC\,V^{-1}}$. The relative atomic masses of radium and bromine atoms are 226 and 80 respectively. The gold leaf rises and falls every 85 seconds.

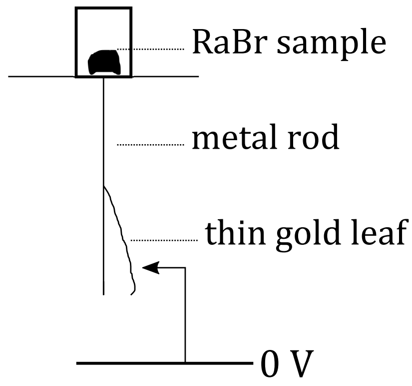


Figure 1

(i) Calculate the number of beta decays which occur for the flexible gold leaf to rise and make contact with the electrode.

(ii) Calculate the activity of the sample.

(iii) Hence determine the half-life of Ra-226.

(iv) What is the average current flow along the contact to earth during the first day?

(v) Comment on whether a similar gold leaf apparatus could be used to measure the initial current flow from the sphere used in part (c).

(vi) What would be the effect on the rise and fall of the gold leaf of replacing the radium bromide sample with a sample having the same initial activity but made from the isotope Ra-223 with a half life of 11.4 days? Sketch a graph of the rise and fall time, T, of the gold leaf over a period of 50 days.

(vii) If this apparatus with the original Ra-226 sample was used as a clock, how long would it take for the clock to have slowed by one hour in a correctly measured 24 hour period?

(8)

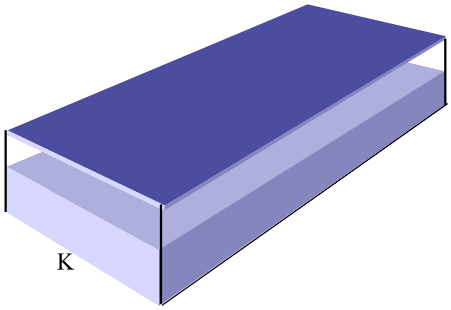
[25 marks]

Question 3

This question is about dynamics in nautical examples.

a) An oil tanker as illustrated in **Figure 2a** will typically have a length of $272\,\mathrm{m}$, a breadth of $48\,\mathrm{m}$ and a height from the bottom of the ship to the deck surface of $24\,\mathrm{m}$. The mass of the ship alone is $22\,000\,\mathrm{tonnes}$. A typical full load would be $158\,000\,\mathrm{tonnes}$ of crude oil with a density of $900\,\mathrm{kg\,m^{-3}}$.
We model the tanker as a closed cuboid tank carrying oil at the bottom, of uniform cross-section, as shown in **Figure 2b**.

(a) Oil tanker. credit: Photograph: Baird Maritime



(b) Simple model of the tanker

Figure 2

(i) Calculate the distance from the water line to the deck when the tanker is loaded with a full tank that occupies the lower part of the ship as shown in **Figure 2b**. Take the density of sea water to be $1025\,\mathrm{kg\,m^{-3}}$.

(ii) Calculate the height of the centre of mass of the loaded ship above the base centre point, **K**, shown in **Figure 2b**.

(iii) The ship lists to one side to the point where water just reaches the deck, as illustrated in **Figure 3**. Calculate the angle to the vertical θ, where θ is a small angle.
[The buoyancy force acts at the centre of mass of the displaced water.]

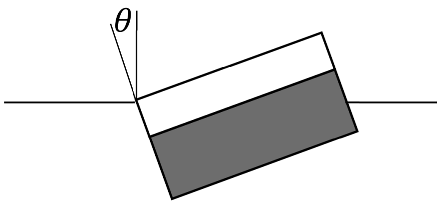


Figure 3: When the ships tilts, we assume the oil remains stationary inside due to internal baffles and walls

(iv) Determine the resultant turning moment acting on the ship.

(8)

b) A **capstan** (a rotating cylinder) on the ship pulls up a uniform heavy chain from the dockside by means of a much lighter thin rope of negligible mass, as in the photograph in **Figure 4**.

(i) The chain is pulled up from the ground at a constant speed v. It has a mass per unit length of λ. If the length of heavy chain lifted from the ground and suspended from the rope is taken to be x, find expressions for the force, F, and the power, P, supplied by the ship's capstan in terms of x, λ, v and g.

(ii) Find an expression for the upward contact force R from the dockside, acting on the chain whilst it is being pulled up, taking the total length of chain to be L.

Figure 4

(6)

c) The heavy chain, with an effective cross-sectional area of A, stretches under its own weight. Find an expression for the extension of the chain e, as a function of its suspended length x. The heavy chain has a mass per unit length μ, a Young's modulus of E, and the gravitational field strength is g. The cross-sectional area of the chain is assumed to remain constant.

(5)

d) A strong rope can be used to pull on a large ship by wrapping the rope around a capstan (a cylinder) and holding the loose end of the rope with a small force. The friction of the rope on the capstan acts as a force multiplier. If a rope is in contact with a cylinder with friction acting, as shown in **Figure 5**, in which the angle θ is small.

(i) Sketch a fully labelled diagram to show the normal component of the tension acting on the capstan from the rope.

(ii) Obtain an expression relating the tension in the rope T, ΔT, θ and the coefficient of friction μ.
The coefficient of friction is given by; frictional force $= \mu \times$ normal reaction force of the capstan.

(iii) By integrating your result relating ΔT and T, obtain the relation between the tensions on the ends of the rope for large angles θ.

(iv) Approximately how many turns of a rope on a capstan for which the coefficient of friction $\mu = 0.3$ would be required if you were to balance a person's weight of $\approx 600\,\mathrm{N}$ with a weight equivalent to the "weight of the Earth" (6×10^{25} N)?

(6)

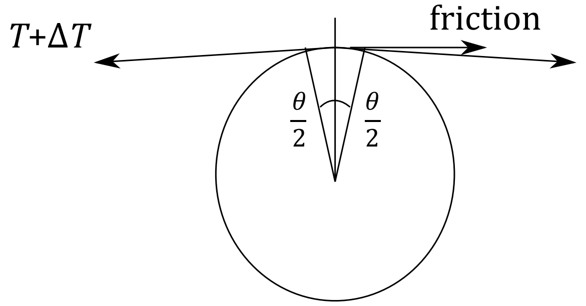


Figure 5

[25 marks]

Question 4

The change in speed of a wave in a medium has many important aspects, some of which are illustrated in this question.

a) A prism of apex angle A and refractive index n has a monochromatic beam of light incident on a face as shown in **Figure 6**.

(i) Sketch a large diagram of the prism and draw the path of a ray passing through the prism. Mark on the angles of incidence for each face and the angles of refraction for each face for the ray, following the path of the ray with the angles in path order, $\theta_1, \theta_2, \theta_3$ and θ_4.

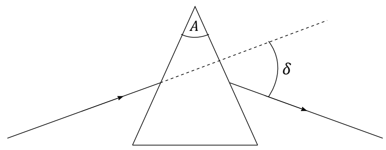


Figure 6

(ii) Obtain an expression for the angle of deviation, δ, between the incident ray and the emerging refracted ray in terms of the incident angle of refraction θ_1, the angle of refraction of the ray emerging from the second face, θ_4 and A.

The minimum angle of deviation, δ_{min}, occurs when the ray passes through the prism symmetrically. Then $\theta_1 = \theta_4$ and $\theta_2 = \theta_3$.

(iii) By considering a symmetric ray passing through the prism, obtain an expression for the the refractive index of the prism in terms of δ_{min} and A.

(iv) For a prism with apex angle $A = 60°$, what is the difference in the angle of deviation for red and blue light, for which the refractive indices are respectively $n_r = 1.604$ and $n_b = 1.620$? Assume each ray passes through the prism with minimum deviation.

(v) If the emerging rays are focused on to a screen by a lens with a focal length of $30\,\mathrm{cm}$, what will be the linear separation of the images on the screen?
[parallel rays passing through the centre of a lens are undeviated in direction but will be focused on the screen a focal length away]

(vi) Show that when the apex angle of the prism A is small, then the refractive index can be given by $\delta = A(n-1)$.

(vii) A glass prism of apex angle $6.0°$ and of refractive index $n_1 = 1.50$ is held with its apex downwards alongside another prism of apex angle $4.0°$ which has its apex upwards. A parallel beam of monochromatic light is at normal incidence to the first prism and emerges from the second, parallel to the original beam, but displaced. Calculate the refractive index of the second prism.
The thin prism approximation of (vi) can be used for both prisms.

(10)

b) In **Figure 7**, a parallel beam with a wavefront of width w passes symmetrically through the isosceles prism. If t_1 is the time taken for the light to travel the path $\mathbf{P_1VP_2}$, and t_2 is the time taken to travel $\mathbf{Z_1Z_2}(= L)$, obtain an expression for the ratio of the times $\dfrac{t_1}{t_2}$ in terms of the incident angle θ_1, A and n. **(3)**

c) A thin lens has the characteristics of a thin prism. Using the result quoted earlier for a thin prism, $\delta = A(n-1)$, and the lengths u and v with corresponding small angles α and β shown in **Figure 8**, in which the top half of the lens is shown,

(i) obtain an expression relating u, v, A, n and the radius of the lens d.

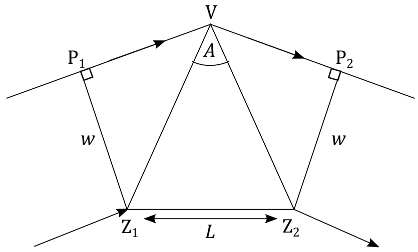


Figure 7

(ii) If a thin lens has a diameter of $5.0\,\mathrm{cm}$, and $n = 1.5$, what is the value of the apex angle A for the lens with a focal length of $40\,\mathrm{cm}$?
(The focal length is the distance from the lens where a set of parallel rays entering the lens would cross the axis.)

(3)

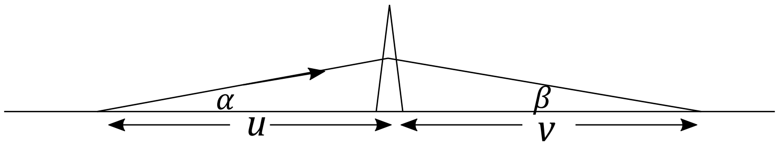


Figure 8

d) A $60\,\mathrm{m}$ tank of water has a stroboscope at one end producing white light in equal time on/off flashes at a frequency f_0. The refractive index of water for red light is 1.33 and for blue light it is 1.35.

Estimate the maximum flash frequency at which light pulses can enter the tank and emerge from the far end as separate flashes.

(3)

e) From the Earth's surface, a star can be observed through a telescope. However, the angle of the star measured from the vertical is shifted due to refraction by the Earth's atmosphere, as indicated in the model in **Figure 9**. The true position of the star is at angle α to the vertical, and the observed angle is at α_0 to the vertical, with the small difference in angle being given by r, such that $\alpha = \alpha_0 + r$. It should be assumed that r is a small angle.

(i) Obtain an expression relating α and α_0 for the refracted light.

(ii) Hence find an expression for r in terms of $\tan\alpha_0$ and n_0, the refractive index of the atmosphere at the Earth's surface.

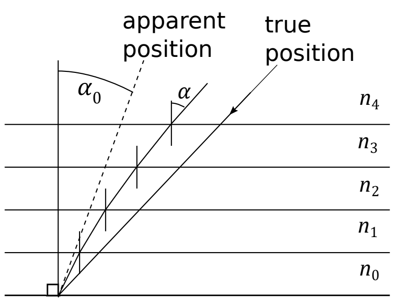


Figure 9: The atmosphere is treated as distinct horizontal layers of refractive index from $n_0, n_1, \ldots$ to n_n at the top.

(6)

[25 marks]

Question 5

This question is about circular motion and gravity.

a) A hump-back bridge has a circular cross section. It crosses a river of width $18\,\mathrm{m}$ and the height of the centre of the bridge above the surrounding ground is $4.0\,\mathrm{m}$. What is the maximum speed at which a car can drive over the bridge whilst still remaining in contact with it?
Hint: if two chords of a circle, AB and CD meet at a point S then AS·SB = CS·SD

(3)

b) The Sun orbits the centre of the Milky Way galaxy at a radius of $30\,000$ light years, and a period of orbit about the galaxy of 200 million years. Assuming that the mass distribution of matter in the galaxy is concentrated mainly in a central uniform sphere and the Earth lies in one of the arms, as illustrated schematically in **Figure 10**,

(i) obtain an expression for the period of orbit T of the Sun in terms of its radius of orbit, r, G and M_g, the mass of the galaxy.

(ii) Determine a value of the mass of the galaxy.

(iii) If the Sun represents the mass of an average star, estimate the number of stars in the Milky Way.

(3)

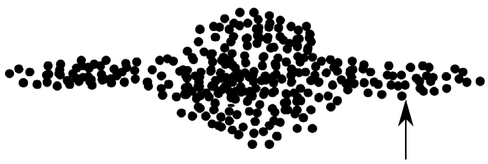


Figure 10: Schematic of the Milky Way galaxy, viewed in the plane, with the Sun along one 'arm', marked by the arrow.

c) A satellite orbits the spherical Earth of radius R_E in a circular orbit of radius r.

(i) Write down an expression for the speed v of the satellite in terms of r, G and M_E, the mass of the Earth, explaining the physics needed to obtain your expression.

(ii) Evaluate the speed of a satellite in a low orbit, if it was just grazing the Earth.

(iii) Calculate the period of the satellite orbit.

(iv) The satellite instead orbits at a height $h \ll R_\mathrm{E}$ above the surface of the Earth. Obtain an expression for the fractional change $\dfrac{\Delta T}{T}$ of the orbital period T in terms of h and R_E. Hence what is the period of orbit of the satellite orbiting at a height of $200\,\mathrm{km}$?

(v) Sketch a graph of the period of orbit T against h for $h \ll R_\mathrm{E}$.

(vi) Show that when a large mass m_1 moving at speed u collides elastically with a stationary small mass m_2, the kinetic energy lost by m_1 is $\approx 2m_2u^2$

(vii) When a small satellite of mass m and cross-sectional area A passes through a gas of density ρ it will make numerous elastic collisions with molecules of the gas. Obtain an expression for the rate of loss of energy of the satellite in terms of ρ, A and v.

(viii) An Earth satellite of mass $20\,\mathrm{kg}$ is in a circular orbit at a height small compared to the radius of the Earth. If air resistance causes its total energy to decrease by $10\,\mathrm{kJ}$ per revolution, what is the fractional change in its speed per revolution?

(ix) By considering an expression for total energy of a satellite in orbit, show whether the speed increases or decreases.

(x) By simply using values for the period and speed of an Earth orbiting satellite in a grazing orbit, if the cross-sectional area of the satellite is $2.4\,\mathrm{m}^2$, estimate the density of the atmosphere for this rate of energy loss.

(14)

d) In the previous parts of this question, we assumed that the mass of the Earth was very much larger than the satellite. In the remainder of this question, we relax this approximation.

Many stars are found as part of binary systems, consisting of two stars of unequal mass in circular orbits about one another.

(i) Draw a diagram of the orbits of such a binary system labelling the positions of the two stars at two successive times in their orbits.

(ii) Suppose the stars have mass m and km with $k > 1$, what are their relative orbital radii?

(iii) One day the more massive star explodes in a supernova. After the explosion, the masses of the stars are now equal, and the exploding material has moved to a great distance away. Describe using diagrams the motion of the star system if the binary system were to remain bound. Assume the supernova causes the larger star to lose mass in a spherical symmetrical manner.

(iv) Find the critical value of the initial mass ratio k, above which the binary system becomes unbound after the explosion described in (iii).

(5)

[25 marks]

Question 6

This question is about atomic structure and electric fields.

The sodium chloride crystal has a face centred cubic structure and we shall consider a unit cell (smallest repeating unit) which is in the octahedron shown shaded in **Figure 11**. For the purposes of this question we will consider the upper left octahedron, with the (blue/grey) circles at the corners to be the chlorine ions, and the (black) central ions to be sodium.

The ions in the crystal lattice are held in place by electrostatic forces, and the ions are considered to be point charges. The charge on a sodium ion is $+1e$; the charge on a chlorine ion, $-1e$. The distance a, between any two adjacent chlorine ions is $0.564\,\mathrm{nm}$.

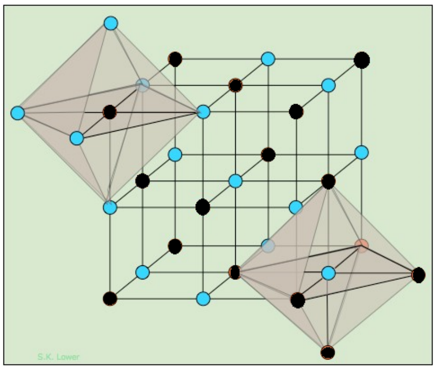


Figure 11:
credit: Figure attributed to S K Lower, https://doclecture.net/1-49591.html

a) What is the separation between nearest sodium and chlorine ions? **(1)**

b) Calculate the electric potential energy of the unit cell, considering it to be the building block for the solid, so that you can take its overall charge to be zero and assign an appropriate charge to each chlorine ion. If, in our very simple model, we ignore the interaction between chlorine ions, estimate the energy released when 1 mole of NaCl is formed. **(3)**

c) Now consider the octahedron shown in isolation. One sodium ion surrounded by six chlorine ions, the latter which we shall consider to be in fixed positions. If the sodium ion is displaced by a small distance x towards one of the chlorine ions, it will experience a resultant force towards its equilibrium position. Using suitable approximations when $x \ll a$, determine:

(i) An expression, in terms of the electronic charge e, the unit cell dimension a, the small displacement x, and the constants π and ϵ_0, for the resultant force acting on the sodium ion.

(ii) Estimate the frequency of oscillation, assuming it approximates to simple harmonic motion, and the mass of the ion to be $23\,\mathrm{u}$. **(6)**

One surface of the sodium chloride crystal looks like **Figure 12**, and this surface can be investigated using electron diffraction.

d) A beam of electrons accelerated through a potential of $100\,\mathrm{V}$ is directed normally on to the surface. The electrons scatter back elastically from the surface layer of ions. By considering the electrons to behave as waves diffracted by the surface, determine

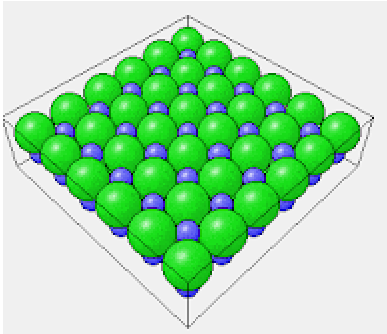


Figure 12:
credit: http://physicsforeach.blogspot.com/2012/09/oth

(i) an expression for the angle of interference maxima (relative to the normal) along the principle directions shown by the box edges in the diagram. Use the standard notation of: a for the separation of two adjacent scatterers, n for order of diffraction, θ for diffraction angle of the interference maxima, m for the mass of the electron, V for the potential, and h for Planck's constant.
[Note: it makes no difference if the beam originates on the same side of the surface when it undergoes scattering.]

(ii) Calculate the angle of diffraction for the first and second orders.

Now consider the electron to be a particle with a momentum p.

(iii) For the first order diffracted beam, draw a diagram using momentum vectors to show the change in momentum of the electron during collision with the surface. Record the angles between the vectors on your diagram.

(iv) Calculate the direction (relative to the surface) and magnitude of the momentum change. **(8)**

e) The sodium atom must be ionised to form sodium chloride. The first ionisation energy of sodium is $496\,\mathrm{kJ\,mol^{-1}}$. By considering the outermost electron to lie on the surface of a sphere, within which the remaining charge of the atom is contained, estimate the radius R of the atom. **(2)**

f) Now consider the sodium atom ($Z = 11$) to consist of a point like nucleus which contains all of the positive charge of the atom, and the electrons each of charge $-e$, evenly distributed within the atom, giving an average charge density of σ. The radius of the atom is R.

(i) Determine an expression for the electric field strength within the atom as a function of distance x, from the centre of the atom.

(ii) Subsequently find an expression for the electric potential V within the atom, as a function of distance x from the centre of the atom.

(iii) Finally, by considering the atom to consist of concentric spheres of constant charge density, find an expression for the work done assembling the atom, and provide a numerical estimate of the energy required to turn one mole of sodium into a fully ionised gas (a plasma) of electrons and nuclei.
[Use your result from part (e) for the radius of the atom.]

(5)

[25 marks]