BPhO Round 1

Section 2

12th November 2021

This question paper must not be taken out of the exam room

Instructions

Time: **5 minutes reading time (NO writing)** and then **1 hour 20 minutes** for writing.

Questions: Only **two questions** out of the four questions in *Section 2* should be attempted.

Each question contains independent parts so that later parts should be attempted even if earlier parts are incomplete.

Working: Working, calculations, explanations of the physics and **diagrams**, properly laid out, must be shown for full credit. The final answer alone is not sufficient. Writing must be brief but clear. If derivations are required, they must be mathematically supported, with any approximations stated and justified.

Marks: Students are recommended to spend about 40 minutes on each question. Each question in *Section 2* is out of 25, with a **maximum of 50 marks from two questions** only.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

Solutions: Answers and calculations are to be written on loose paper **ON ONE SIDE ONLY** (these will be scanned). Students should ensure that their **name** and their **school/college** are clearly written on each and every answer sheet. Number each question clearly **and** number your pages at the top.

Setting the paper: There are two options for sitting BPhO Round 1:

a. *Section 1* and *Section 2* may be sat in one session of 2 hours 40 minutes plus 5 minutes reading time (for *Section 2* only). *Section 1* should be collected in after 1 hour 20 minutes and then *Section 2* given out.

b. *Section 1* and *Section 2* may be sat in two sessions on separate occasions, with 1 hour 20 minutes plus 5 minutes reading time allocated for *Section 2*. If the paper is taken in two sessions on separate occasions, *Section 1* must be collected in after the first session and *Section 2* handed out at the beginning of the second session.

Important Constants

Constant	**Symbol**	**Value**
Speed of light in free space	c	$3{:}00 \quad 10^{8}\,\mathrm{m\,s}^{\;1}$
Elementary charge	e	$1{:}602 \quad 10^{\;19}\,\mathrm{C}$
Planck constant	h	$6{:}63 \quad 10^{\;34}\,\mathrm{J\,s}$
Mass of electron	m_{e}	$9{:}110 \quad 10^{\;31}\,\mathrm{kg}$
Mass of proton	m_{p}	$1{:}673 \quad 10^{\;27}\,\mathrm{kg}$
Mass of neutron	m_{p}	$1{:}675 \quad 10^{\;27}\,\mathrm{kg}$
atomic mass unit	u	$1{:}661 \quad 10^{\;27}\,\mathrm{kg} = 931{:}5\,\mathrm{MeV\,c}^{\;2}$
Gravitational constant	G	$6{:}67 \quad 10^{\;11}\,\mathrm{m}^{3}\,\mathrm{kg}^{\;1}\,\mathrm{s}^{\;2}$
Earth's gravitational field strength	g	$9{:}81\,\mathrm{N\,kg}^{\;1}$
Permittivity of free space	$''_{0}$	$8{:}85 \quad 10^{\;12}\,\mathrm{F\,m}^{\;1}$
Avogadro constant	N_{A}	$6{:}02 \quad 10^{23}\,\mathrm{mol}^{\;1}$
Gas constant	R	$8{:}3145\,\mathrm{J\,K}^{\;1}\,\mathrm{mol}^{\;1}$
Mass of Sun	M_{S}	$1{:}99 \quad 10^{30}\,\mathrm{kg}$
Radius of Earth	R_{E}	$6{:}37 \quad 10^{6}\,\mathrm{m}$
Specific heat capacity of water	c_{w}	$4180\,\mathrm{J\,kg}^{\;1}\;\;\mathrm{C}^{\;1}$

$$T_{(\mathrm{K})} = T_{(^\circ\mathrm{C})} + 273$$

$$\text{Volume of a sphere} = \tfrac{4}{3}\;\; r^{3}$$

$$e^{x} \qquad 1 + x + ::: \qquad\qquad x \quad 1$$

$$(1 + x)^{n} \qquad 1 + nx \qquad\qquad x \quad 1$$

$$\frac{1}{(1 + x)^{n}} \qquad 1 \quad nx \qquad\qquad x \quad 1$$

$$\tan \qquad \sin \qquad\qquad \text{for} \quad 1$$

$$\cos \qquad 1 \quad \frac{2}{2} \qquad\qquad \text{for} \quad 1$$

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Section 2 — attempt two questions only

Question 2

This question is about waves on wires and in tubes.

a) A uniform wire of length ℓ and mass per unit length, μ, is stretched between two fixed points . A point close to one end is plucked and a pulse travels along the wire at speed v, as shown in **Fig. 1**, to reflect off the right hand fixed end.

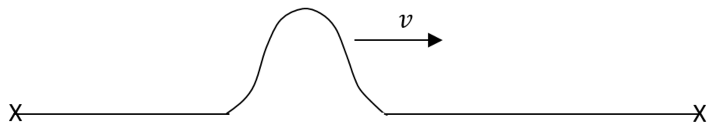

Figure 1: Pulse travelling at speed v along a wire under tension.

(i) Sketch a diagram of the reflected pulse.

(ii) After a second reflection from the left hand end, the pulse would be passing the initial point and travelling in the same direction. At that moment the wire could be plucked again to reinforce the pulse. What is the period of plucking, $t(\ell; v)$ which would accomplish this?

(iii) Calculate the corresponding frequency of plucking the wire, f_1, in terms of v and ℓ.

(iv) If a mechanical vibrator is attached to the wire to produce oscillations of frequency $f_n = nf_1$ (which are the harmonics or multiples of the 1st harmonic, f_1) what resonant frequencies of the wire will be sustained, in terms of integer n, v and ℓ.

(v) The wire is now plucked near one end whilst a finger is lightly touching the wire at a point 40% of length from one end. Sketch the resultant amplitude of vibration along the wire, and determine the two lowest frequencies heard in terms of f_1.

(vi) The speed of transverse waves along a stretched wire wave is given by $v = \sqrt{\frac{T}{\mu}}$, with T the tension in the wire. What would be the tension required in order to sustain the fifth harmonic, f_5, of frequency $162\,\mathrm{Hz}$ in the wire if its length is $2.3\,\mathrm{m}$ and mass is $42\,\mathrm{g}$.

(6)

b) In this example we hang the cable vertically and use its own weight to provide the tension. A massive, uniform cable of length L and mass M hangs with its upper end fixed to a support and the lower end free. The extension is negligible.

(i) What is the tension at a distance x below the support?

(ii) The cable is shaken slightly at the bottom end. Using the information in part (a)(vi), calculate the ratio of the speeds of the pulse on the cable at points $1/4$ of its length from the top end and $1/4$ of its length from the bottom end.

(iii) The cable is now plucked at the top end as in **Fig. 2** and the pulse travels down the cable. Write an expression for the speed of a pulse travelling down the cable in terms of x, L and g.

(iv) Now obtain an expression for the time taken, t_{top} for a pulse to travel a distance x down the cable from the support at the top end.

(v) At the same moment that the cable is plucked at the top end, a ball is dropped from rest from the same starting height. At what value of x in terms of L will the ball overtake the pulse traveling down?

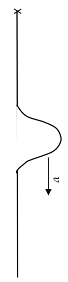

Figure 2: Pulse travelling down a vertical hanging cable.

(8)

c) The timing of pulses can be used in GPS systems.
A stationary receiver is at a point on the line between two radio transmitters which lie on a West-East line, as in **Fig. 3**. Each transmitter contains a highly accurate clock and broadcasts a precise $1\,\text{MHz}$ electromagnetic wave in pulses. After each pulse, a signal is sent with the exact time at which that signal was sent. The radio transmitters are $600\,\text{km}$ apart. At the receiver, pulses are received from both transmitters. The pulse from the Western transmitter is received $1542\,\text{cycles}$ of the $1\,\text{MHz}$ wave before the pulse from the Eastern transmitter is received. The pulse from the Western transmitter was sent $0{:}000\,484\,\text{s}$ before the pulse from the Eastern transmitter.
How far is the receiver from the Western transmitter? **(2)**

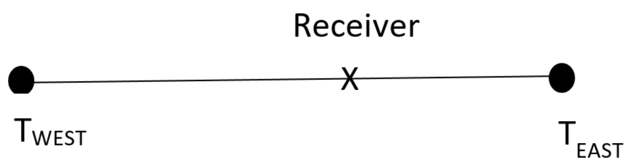

Figure 3: Two transmitters and a receiver lying along a W-E line

d) A resonant tube of length ` is closed at one end and has a small loudspeaker close to the open end as in **Fig. 4**. An a.c. signal of frequency f is supplied to the speaker, and adjusted so the tube resonates at its lowest frequency, f_1. This longitudinal sound wave has a displacement node at the closed end and an antinode at the open end.

(i) Sketch three graphs of the amplitude of the standing wave along the tube, for the lowest three harmonics that can be heard, as the speaker frequency f is steadily increased.

(ii) Now in dotted lines but on the same axes as already drawn in part (i), sketch three graphs of the corresponding pressure variation of the air with position along the tube.

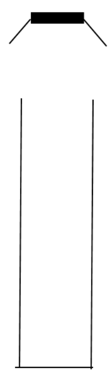

Figure 4: Speaker above the open end of a closed tube.

(2)

e) The speed of sound in air is given by the equation $v_{\mathrm{gas}} = \sqrt{\frac{P}{\ }}$ where   is a numeric constant, P is the pressure and   is the density of the air.

(i) In a $20\,\mathrm{m}$ tall vertical tube closed at one end, by how much is the pressure of air greater at the bottom of the tube relative to the top?

(ii) Why is it that the speed of sound does not vary with the depth of the air in the tube?

(iii) However, the speed does depend upon the absolute temperature of the air. Obtain an expression for the dependence on the speed of sound in terms of the constant , the gas constant R, the absolute temperature T and the kg molar mass M_{u} in $\mathrm{kg\,mol}^{-1}$.

(iv) In a tube closed at one end, the antinode at resonance is located a (small) fixed distance ″ beyond the open end of the tube. This length is known as the *end correction* and is illustrated in **Fig. 5**.

A tube closed at one end, of length $52.0\,\mathrm{cm}$, and filled with air at $20.0\ \mathrm{C}$, resonates at its first harmonic at $156\,\mathrm{Hz}$. A second, shorter tube alongside, which has the same *end correction*, ″, is filled with warm air at $35\ \mathrm{C}$ and resonates at a slightly different frequency, producing beats with the first tube at $4.8\,\mathrm{Hz}$. What is the length of the second tube?

At $20\ \mathrm{C}$ the speed of sound in air is $343\,\mathrm{m\,s}^{-1}$.

Atmospheric pressure is $1.01 \times 10^5\,\mathrm{Pa}$.

Density of air at atmospheric pressure and $15\ \mathrm{C}$ is $1.225\,\mathrm{kg\,m}^{-3}$.

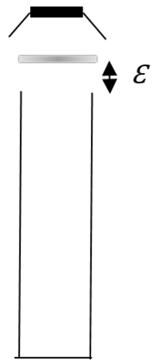

Figure 5: The *end correction*, ″ for an antinode located above the open end of the tube.

(7)

[25 marks]

Question 3

This question is about the properties of some electrical circuits.

a) The circuit in **Fig. 6** shows a chain of identical resistors of value $4{:}0$. A current of $1{:}0$ A flows through the final resistor.

(i) What is the potential difference across AB?

(ii) What is the equivalent resistance of the chain?

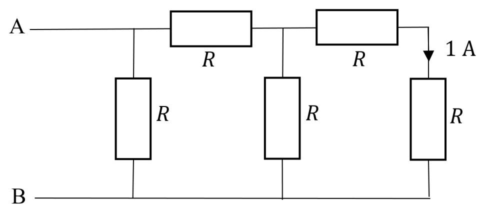

Figure 6

(2)

b) A rechargeable cell, modelled as an emf E in series with an internal resistance r, has a current of $0{:}5$ A flowing when a charging potential of $2{:}5$ V is applied. When discharged through a $7{:}6$ resistor, a current of $0{:}25$ A flows.
Determine the emf and internal resistance of the cell. **(3)**

c) A cell of emf E is connected to a resistor R that is in series with a pair of resistors R_A and R_B as shown in **Fig. 7**.

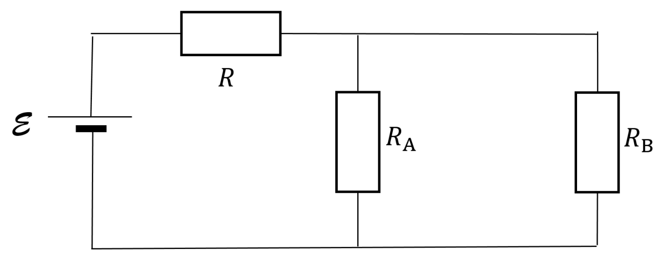

Figure 7

(i) For the two resistors R_A and R_B in parallel, show that an expression for the equivalent resistance, R_{eq} can be written in the form $\frac{R_A R_B}{R_A + R_B}$.

(ii) If a current I was flowing through R,

i. what fraction of I would flow through R_B, and

ii. give an expression for the current I_B through R_B in terms of $E; R; R_A$ and R_B.

(2)

We can use this information to analyse a circuit with more than one source of emf.

(iii) A simple circuit consisting of two cells in series with a resistor$R$ is shown in **Fig. 8**.

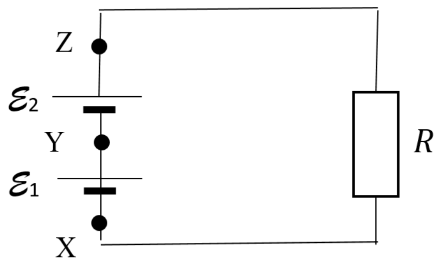

Figure 8: Two cells in series with a resistor.

i. If cell E_2 was removed and points Y and Z joined, what would be the current I_1 now in the circuit?
ii. If cell E_1 was removed instead, and points X and Y joined, what would be the current I_2 now in the circuit?

(1)

If the two cells are both in the circuit, you can see how each source of emf contributes to the current flow. They can be added linearly to give the total current flow.
The circuit shown in **Fig. 9** consists of two cells E_1 and E_2 connected to resistors $R_1; R_2$ and R_3.

(iv) Remove cell E_1 and connect the break in the circuit. Using the notation of this circuit, write down the current flow I_3^{0} through R_3.

(v) Replace cell E_1 and remove cell E_2, connecting the circuit at the break. Write down the current flow I_3^{00} through R_3.

(vi) By linearity, $I_3 = I_3^{0} + I_3^{00}$. Write down an expression for I_3 in terms of $E_1; E_2; R_1; R_2$ and R_3.

(3)

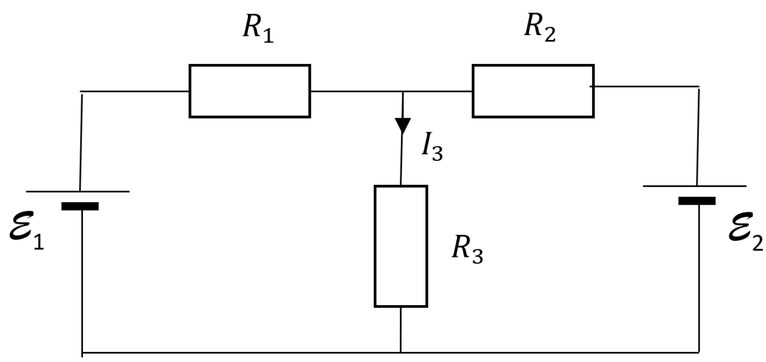

Figure 9: Two cells and three resistors in a circuit.

(vii) This circuit can also be viewed as two cells in parallel, each with an internal resistance, connected to an external load resistor R_3. By considering values of R_3 or otherwise, show that that the cells in parallel behave as a source of emf given by

$$\frac{\frac{E_1}{R_1} + \frac{E_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

and internal resistance

$$R_{\mathrm{int}} = \frac{R_1 R_2}{R_1 + R_2}$$

(viii) Using this result, if $E_1 = 50\,\mathrm{V}; E_2 = 60\,\mathrm{V}; R_1 = 10$ and $R_2 = 15$, calculate

i. the value of R_3 that will dissipate maximum power in R_3 (this occurs when the load resistor is equal to the internal resistance of the supply), and

ii. determine the value of the maximum power in R_3.

iii. If the resistance of R_3 is increased, one cell will begin to charge the other. At what value of R_3 does this begin to occur?

(6)

d) Circuits with non-linear components are of great importance. The following circuit of **Fig. 10** resembles ones just analysed, but its behaviour is different. The circuit is designed to prevent damage to the device represented by the $10\,\mathrm{k}$ load resistor when the circuit is suddenly connected to a $2000\,\mathrm{V}$ d.c. supply. The component S is a voltage suppressor, which has a V–I characteristic given by $I = kV^2$, where V is the voltage across S when current I flows through it. The constant k has a value $k = 1{:}0 \quad 10^{7}\,\mathrm{A\,V}^{2}$.

(i) What is the value of the current in the $10\,\mathrm{k}$ load resistor when operating normally?

(ii) A surge in the supply voltage causes the current in the load resistor to double from its normal value. What is the value of the surge voltage from the supply?

(8)

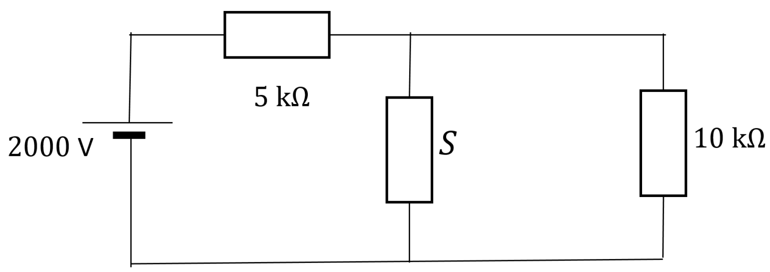

Figure 10: A circuit with a voltage suppressor S and a $10\,\mathrm{k}$ load resistor.

[25 marks]

Question 4

This question is about electric fields.

a) A capacitor C consists of two parallel conducting plates, of large area A, separated by a distance d. A charge $+Q$ is applied to one plate and Q to the other, as shown in **Fig. 11**.

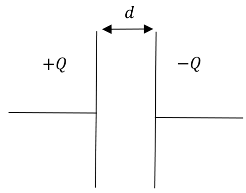

Figure 11: A parallel plate capacitor with charges $+Q$ and Q on the plates.

(i) Sketch three diagrams, showing the electric field due to one plate alone, the electric field due to the other plate alone, and the resultant electric field when both plates are in place parallel to each other. Neglect edge effects where the field may be non-uniform at the edges of the plates.

(ii) Both plates are now positively charged, each with $+Q$ instead. Sketch the three corresponding diagrams of the fields as in part (i).

(iii) The energy of a $+Q$ and Q charged capacitor can be written in terms of Q and C. Given $C = \frac{{}_0A}{d}$, and considering the work done in separating the charged plates by d, derive an expression for the force F acting between the plates in terms of $Q;\ {}_0$ and A.

(iv) Write down an expression for the field strength E between the plates in terms of ${}_0$ and , where $= \frac{Q}{A}$.

(v) If the area of the plates is doubled, by what factor is Q changed to maintain the same force?

(vi) A thunder cloud forms over a large area of still water. What is the electrostatic surface charge density on the water if it rises under the cloud to a height $1{:}5\,\mathrm{mm}$ above its previous level? **(8)**

b) A point charge Q produces a radial electric field, E at distance r from the charge. The same field strength is obtained at the surface of a sphere of radius r if the charge Q is spread uniformly over the surface, with an area charge density $= \frac{Q}{4\ r^2}$.

(i) A conducting sphere of radius r is charged to a potential V. Write down an expression for the charge Q on the surface in terms of $V;\ r$ and ${}_0$.

Two spheres A and B of radii a and b respectively are located with a distance d between their centres, with $a; b \quad d$, as in **Fig. 12**. Each sphere is at the same potential V.

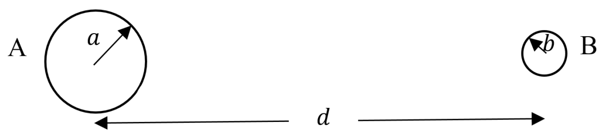

Figure 12: Two charged spheres at the same potential.

(ii) How are the electric field strengths at the surfaces of A and B, E_{A} and E_{B} related to each other?

(iii) An ion of charge e is released from a point $3{:}0\,\mathrm{mm}$ from the surface of B. If B has a radius of $1{:}0\,\mathrm{mm}$ and is at a potentials of $2{:}0\,\mathrm{kV}$, calculate the energy in electron-volts gained by the ion in moving $1{:}0\,\mathrm{mm}$ towards B.

A pear shaped conducting shell with hemispherical ends, as in **Fig. 13** is of circular cross section perpendicular to its longitudinal axis. It is positively charged to a potential $+V$.

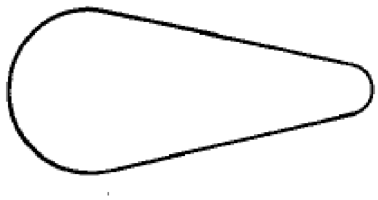

Figure 13: figure

(iv) Copy the diagram and sketch the pattern of electric field lines near the surface of the conductor.

As the potential is raised, the electric field strength at the surface will increase. The air becomes conducting when an electron is accelerated by the E field so that it gains enough energy between collisions to ionise the next atom with which it collides.

(v) If the ionisation energy is $2.5\,\text{eV}$ and the mean free path between collisions is $7.0 \times 10^{-6}\,\text{m}$, calculate the electric field strength which will cause the air to break down.

(vi) The radii of the two ends of the shell are $15.0\,\text{cm}$ and $5.0\,\text{cm}$. What is the maximum potential of the shell such that it will not discharge through breakdown of the surrounding air?

(vii) Sketch the charge distribution over the surface of the shell.

(10)

c) A point charge $+Q$ is located a perpendicular distance d from a large area, thin, earthed, conducting plate, as shown in **Fig. 14**.

(i) Sketch the field lines linking the point charge and the conducting plane.

(ii) Explain

i. why there is a force acting on the plate,

ii. in which direction is the force on the plate, and

iii. why the force vector acting on the plate is along the direction d and passes through the charge Q.

(iii) Consider the idea of removing the positive charge on the left and placing a negative charge on the right of the plate at distance d.

i. Sketch the field lines between the earthed plate and the negative charge.

ii. Now, by considering these field line patterns, with and without the plate present, calculate the attractive force between a $4.0\,\mu\text{C}$ charge located $20\,\text{cm}$ from a large, conducting, earthed plate.

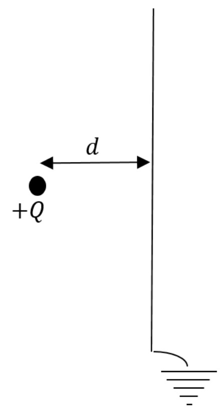

Figure 14: Point charge near a plane

(7)

[25 marks]

Question 5

This question is about springs and forces.

a) (i) A vertical spring of unstretched length ℓ_0 and spring constant k is fixed to a rigid support, as in **Fig. 15**. A mass m is attached and the equilibrium length is then ℓ.

i. The mass is displaced by a small distance x downwards from its equilibrium position. Draw a diagram of the mass and mark on the forces acting.

ii. Referring to your forces diagram, show that the period of oscillation is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

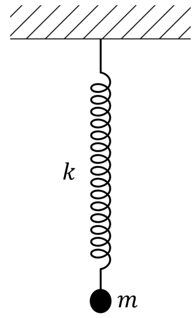

Figure 15: Mass m hanging from a spring.

(2)

(ii) Two identical springs each of unstretched length ℓ_0 are attached to walls a distance $2\ell_0$ apart, and attached at the centre to a mass m, as shown in **Fig. 16**. The mass rests on a smooth, horizontal plane. Each spring has a spring constant k, and can be compressed or stretched.

i. Draw a diagram showing the forces acting on the mass when it is displaced from its equilibrium position by an amount x along the line of the springs.

ii. What would be the period of oscillation of the mass when released?

(2)

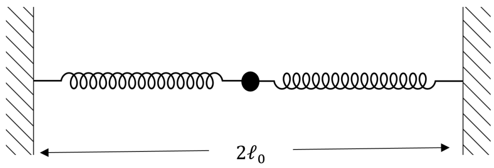

Figure 16: Two springs attached to a mass, which rests on a smooth horizontal plane.

(iii) The walls are now moved apart so that they are separated by distance $4\ell_0$, with the same springs and mass attached, as in **Fig. 17**.

i. On a diagram of the mass, mark on the forces acting on the mass when it is displaced by x from its equilibrium position.

ii. What is the period of oscillation, T, now?

(1)

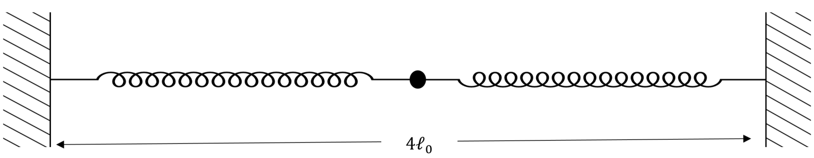

Figure 17: Two identical springs attached to a mass, which rests on a smooth horizontal plane.

(iv) Two springs, each of unstretched length ℓ_0, but with different spring constants k_1 and k_2, are attached to a mass m and are stretched between the two walls a distance $4\ell_0$ apart, as in **Fig. 18**. By making a small displacement x of the mass from equilibrium, obtain the equation for its period of oscillation T.

(2)

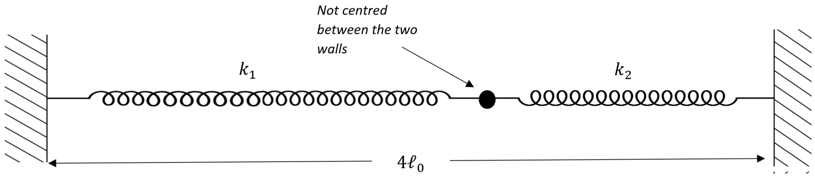

Figure 18: Two springs attached to a mass, which rests on a smooth horizontal plane.

b) (i) Masses m_1 and m_2 resting on a smooth horizontal surface are attached to the ends of a light inextensible string of length r. When set in rotation they rotate about a point on the string at angular speed ω. Obtain an expression for the tension T in the string in terms of m_1, m_2, r and ω.

(ii) A light spring of constant k in compression and extension, has masses m_1 and m_2 attached to the ends, as shown in **Fig. 19**. When disturbed the masses oscillate along the line of the spring. The system can be modelled as two springs with a non-oscillating point. Find a function of the masses $M(m_1, m_2)$, such that the period of oscillation can be expressed in the form $T = 2\pi\sqrt{\frac{M}{k}}$.

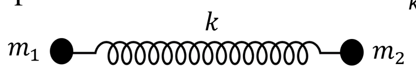

Figure 19: Two masses attached to a spring on a smooth horizontal surface.

(4)

c) The graph shown in **Fig. 20** illustrates the deceleration of a projectile as it hits the ground.

(i) From the graph, using the average decelerations, make simple estimates of the changes in speed in each of the four deceleration regions as it comes to rest, and determine the initial speed of the projectile.

(ii) Similarly, assuming a uniform change in speed with time in each of the four regions of the graph, estimate the depth of penetration of the projectile into the ground.

(5)

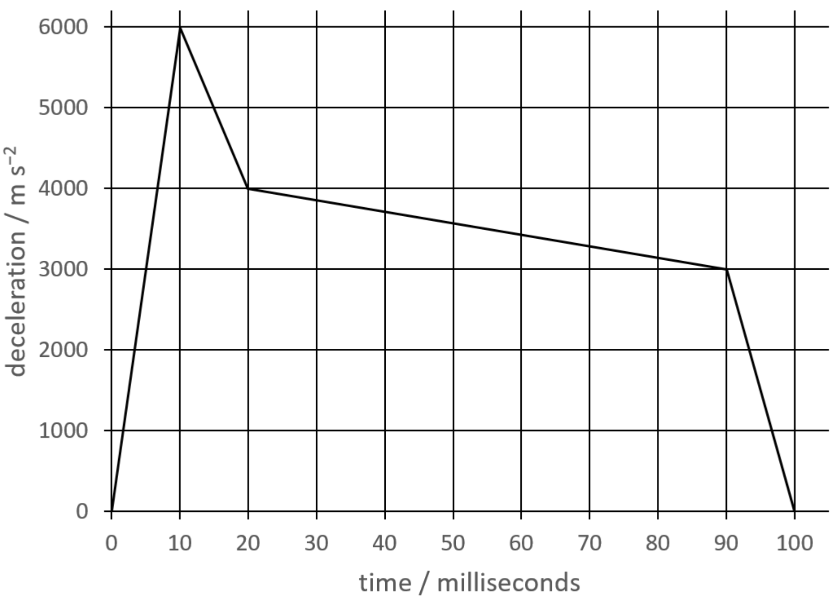

Figure 20: The deceleration profile of a projectile hitting the ground at high speed.

d) Approximations and making a simple model are key to doing physics. Often we require linear behaviour in our model, often associated with a small displacement from equilibrium, and also that the collision time is short compared to any other motion of the system. This we can see in the following example.

In a ballistic experiment, a bullet of mass $m_1 = 10\,\mathrm{g}$ is fired horizontally at a speed of $320\,\mathrm{m\,s}^{\ 1}$ into a rigid container of sand that is fixed to a steel framework, whose sides are represented by very strong, light springs, as shown in **Fig. 21**.
The steel framework together with the box has a mass of $M_2 = 4{:}0\,\mathrm{kg}$ and when tested, the steel frame deflects $5{:}0\,\mathrm{mm}$ when a force of $200\,\mathrm{N}$ is applied horizontally. What is the maximum deflection of the frame when the bullet is fired into the sand?

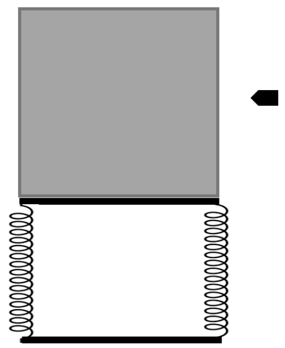

Figure 21: A sand box attached to a stiff steel framework represented by stiff springs, with an approaching bullet.

(4)

e) The approximations mentioned above can also seen in this example.

A steel ball bearing of mass $m_1 = 45\,\mathrm{g}$ is dropped from rest from a height of $h = 2{:}0\,\mathrm{m}$. It rebounds off a steel cylinder of mass $m_2 = 0{:}45\,\mathrm{kg}$ which is supported by a light spring of spring constant $k = 1600\,\mathrm{N\,m}^{\ 1}$, as shown in **Fig. 22**.

If the collision between the ball bearing and the cylinder is elastic,

(i) what would be the speed of the cylinder immediately after impact, and

(ii) what would be the maximum deflection of the spring? You may find it helpful to use the ratio of the masses, $r = \dfrac{m_2}{m_1}$.

(iii) If the collision is not elastic, then we need to have another measurement. Now the cylinder deflects by $12{:}0\,\mathrm{mm}$ from its equilibrium position as a result of the collision. Calculate the height of rebound of the ball.

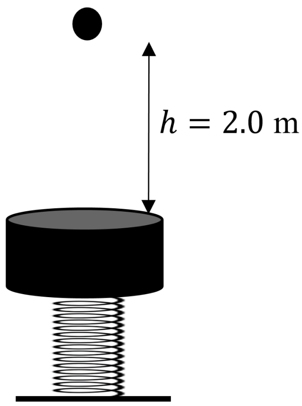

Figure 22: A steel cylinder supported by a light spring.

(5)

[25 marks]

END OF SECTION 2

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