British Physics Olympiad

Paper 3. 2004

Monday 1st March 2004.

Time allowed 3hrs plus 15 minutes reading time.

All questions should be attempted. Question 1 carries 40 marks, the other questions 20 marks each.

Speed of light in free space	c	3.00×10^{8}	$\mathrm{m\,s^{-1}}$
Elementary charge	e	1.60×10^{-19}	C
Mass of a proton (rest mass)	m_p	1.67×10^{-27}	kg
Acceleration of free fall at Earth's surface	g	9.81	$\mathrm{m\,s^{-2}}$
Stefan-Boltzmann constant	σ	5.67×10^{-8}	$\mathrm{Wm^{-2}K^{-4}}$
Standard atmospheric pressure	P_{A}	0.101	M Pa
Avogadro constant	N_{A}	6.02×10^{23}	$\mathrm{mol^{-1}}$
Radius of the Earth	R_{E}	6.37×10^{6}	m
Mass of the Earth	M_E	5.98×10^{24}	kg
Mass of the Sun	M_S	1.99×10^{30}	kg
Radius of orbit of Jupiter	R_{jo}	7.78×10^{11}	m
Mass of the Jupiter	M_J	1.90×10^{27}	kg
Orbital period of the Jupiter	T_J	11.9	years

Mathematics:

$$\int \frac{1}{x^2}dx = -\frac{1}{x} + c\,.$$

Q1

In this question you are asked to make reasoned estimates and assumptions. These must be clearly stated.

a)

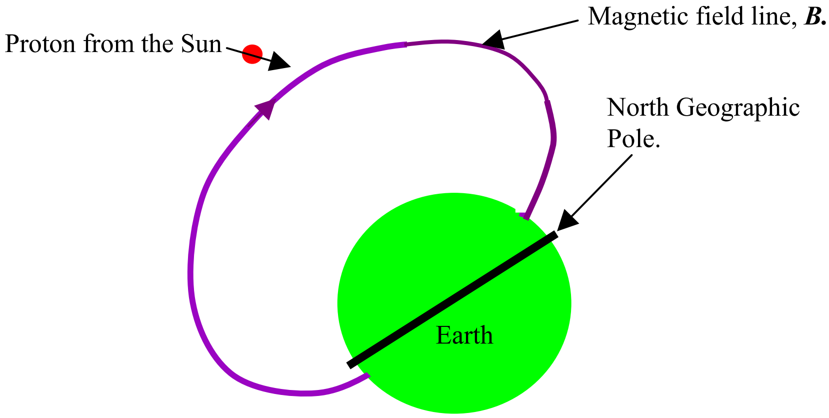
(i) Foolishly I turn on the shower tap when the showerhead is resting on the floor - the flexible pipe thrashes around like a berserk snake. Explain this behaviour. [2]

(ii) The garden hose is 50 m long and connected to a tap. It is then turned on. I partially close the hole emitting the water. The water spurts out faster. Why? [2]

(iii) How could I measure the increased speed of the water – I only have a metre rule? [2]

b)

Figure 1.1



(i) Figure 1.1 shows a cross-section of the Earth and a typical magnetic field line. A proton from the Sun is shown approaching the Earth. Sketch the path of the proton. What effects may be observed when these protons reach the atmosphere of the Earth. [2.2]

(ii) A charged particle, mass m, charge q, travelling in a circle radius r in a magnetic field $\boldsymbol{B}$ at right angles to the plane of the circle emits electromagnetic radiation of frequency f, where f is given by

$$f = \frac{Bq}{2\pi m}.$$

Show that this is equal to the rotational frequency of the charged particle rotating in a plane at right angles to the direction of motion. [2]

(iii) The speed of the proton is 2.1×10^5 m s^{-1} and the magnetic field density, $\boldsymbol{B}$, is 1.0 μT. Calculate the frequency of electromagnetic waves given off. [1]

c) Figure 1.2

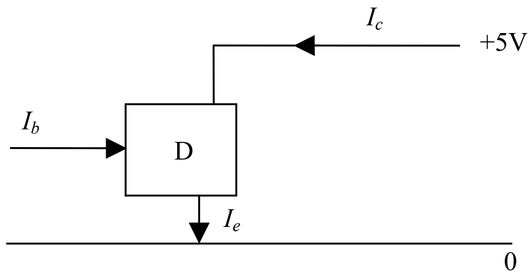


Figure 1.2 shows the circuit diagram needed to test a three terminal device, D. It is found that within the working limits of the device that:

$$I_c = kI_b,$$

where k is a constant. The formula holds only if $I_b > 0$.

(i) Write down a relationship between the three currents. [2]

(ii) How would you modify the circuit so that a small changing p d $\pm\ \delta v_{in}$, (where $\delta v_{in} << 5V$) gave a much larger changing p d, δv_{out}, out? [7]

d) According to the kinetic theory of gases for a perfect gas:

$$p = \frac{1}{3}\rho c^2 ,$$

where p is the pressure of the gas, ρ is the density of the gas and c is the root mean square velocity of the gas molecules.

Find an expression of the rate of leakage of a gas from a small hole, cross-sectional area a, in a cubic container, side l, $l >> \sqrt{a}$. [5]

A cylinder that contains helium has a diameter of 300 mm and a length of 1.5 m. The pressure of the gas in the cylinder is 10 atmospheres above the ambient air pressure. There is a small hole in the piston, diameter 2 mm. Calculate the rate of fall of pressure in the cylinder assuming that the expansion is isothermal. [3]

e)

Figure 1.3,

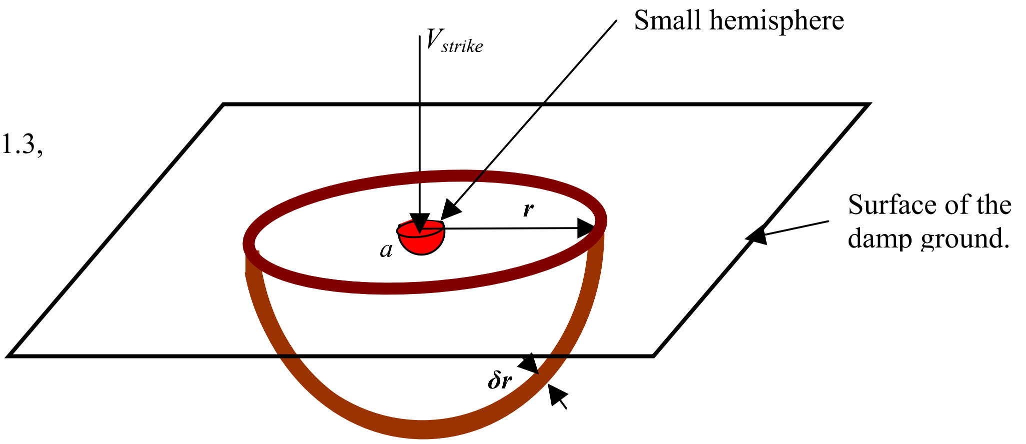


Figure 1.3 shows a diagram of a theoretical model of lightening striking damp ground. The constant potential of the small central hemisphere, radius *a,* is $V_{strike.}$ At a very long distance away, from the region where the lightening strikes the Earth, the potential of a point in and on the Earth is zero. The radius of the hemisphere in Figure 1.3 is r and a thin hemispherical shell, radius r, is of thickness δr. You can assume that the specific resistivity of the damp earth is much less than that of the air and that the current flows radially through the hemisphere. Spherical symmetry is to be assumed on and below the ground.
Sketch graphs (i) the current density σ, (ii) the potential gradient E**,** against r the radial distance from the centre.
Show that for a small change in r, δr, there is a small change in potential δV, given by:

$$\delta V = -\frac{K\rho}{2\pi r^2}\delta r ,$$

where ρ is the specific resistivity of the earth (Ω m) and K is a constant. What is the physical significance of K? [3]
Hence or otherwise find an expression for the potential with distance from the centre of the strike and sketch $V(r)$ against $\boldsymbol{r}$. [4]
In one newspaper report it was stated that lightening struck a field in which there were a herd of cows, and most cows were killed. However in another incident lightening struck a rugby pitch whilst a game was in progress. All, but one, of the players were uninjured, though shocked. The injured man recovered. Suggest a physical reason for the different medical effects in the two incidents. [3]

Total 40

Q2

a) How do the electrical currents in the arms of a network, containing resistors and batteries, alter if the connections to all the batteries are reversed?

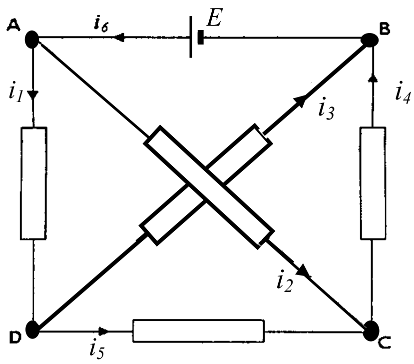
b) In Figure 2.1 all the resistors have resistance R and the cell has an emf of E. The currents indicated in the arms are i_1, i_2, i_3, i_4, i_5, and i_6.

(i) By reversing the polarity of the battery terminals deduce, using a symmetry argument, two relationships between the currents.

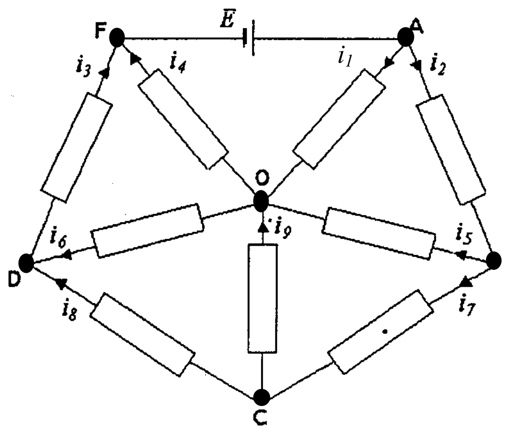
(ii) Show that i_1, $= 0$.

(iii) Determine i_1, i_2 and i_6.

Figure 2.1



c) Figure 2.2



In the circuit shown in Figure 2.2, all resistors have a resistance R

(i) By reversing the polarity of the battery terminals deduce, with appropriate explanation, relations between the currents.

(ii) Determine a value of i_1 in terms of E and R.

(iii) Calculate the total resistance X across the cell in terms of R.

[20]

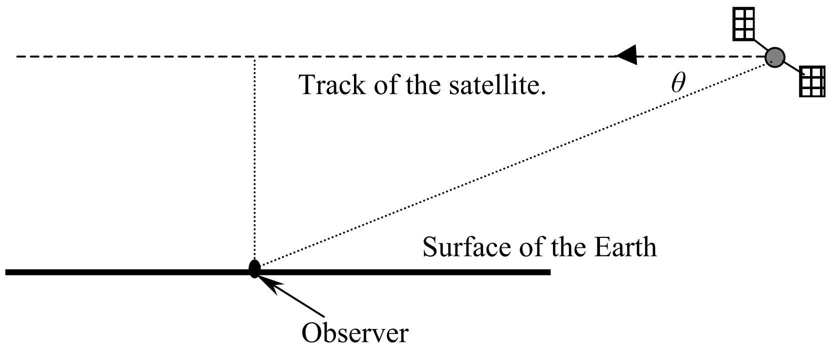
Q3

This question concerns a proposed satellite navigation system.

(a) A satellite orbits the Earth, radius R_E, in a circular polar orbit with a period, T, and a height, h, above the Earth's surface. Find h in terms of T.

(b) Sketch an orbit of the satellite on a rough Mercator projection map of the Earth, and a subsequent orbit at a time τ later. T <24 hrs and $\tau > T$.

Figure 3.1



(c) The satellite sends out a radio frequency f_o. This frequency is monitored by an atomic clock.
Show from first principles that this frequency will be observed by an observer on the Earth as f where

$$f = f_o \pm f_o \frac{v}{c}\cos\theta ,$$

$\boldsymbol{v}$ is the velocity of the satellite and θ is the angle between the satellite's instantaneous direction and the observer, Figure 3.1. Assume $v << c$ where c is the speed of light.

(d) A satellite in low orbit is monitored by an observer at latitude of 0^o and longitude of 0^o. Sketch a graph of Δf, where $\Delta f = f_o \frac{v}{c}\cos\theta$, against time, t, as the satellite (i) approaches (ii) passes overhead and (iii) disappears from the observer.

(e) Find expressions for asymptotes and turning points on your graph.

(f) The observer moves 1^o East. The height of the satellite is 200 km. What is the effect on the frequency of the observed signals as the satellite approaches and recedes next time the satellite passes over the position latitude 0^o and longitude 0^o?

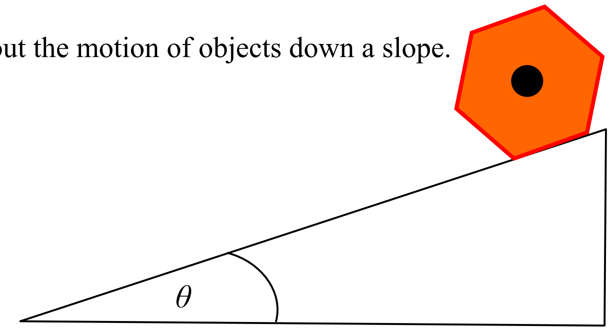
(g) The special theory of relativity predicts that in addition to any of the above effects moving clocks appear to run slow i.e. "$\Delta\Xi_{observed} = \dfrac{\Delta\Xi}{\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}}$" where Ξ is the time variable, $\Delta\Xi$ is a time interval in the inertial frame of the clock and v is the relative velocity of the clock with respect to the observer. What error will have occurred if non relativistic equations are assumed?

[20]

Q4

This question is about the motion of objects down a slope.

Figure 4.1.



(a) Figure 4.1 shows the cross-section of a slope on which a pencil rests. The pencil is hexagonal in cross-section. Find the angle, θ, of the slope, at which the pencil will tend to roll down the slope.

(b) The pencil could slip. Find the value of θ at which the pencil will slide if the coefficient of friction between the surfaces is μ.

(c) Consider a cylindrical object that has a cross-section in the form of a polygon with N equal sides. Find the algebraic condition such that if μ is greater than a certain value the object will roll rather than slip.

(d)

Figure 4.2

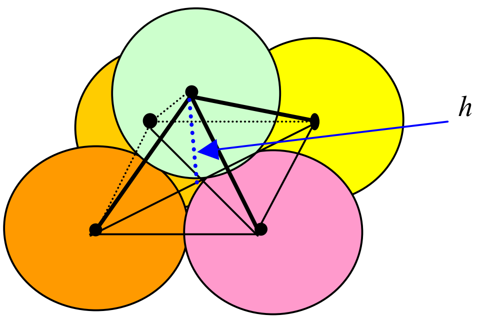
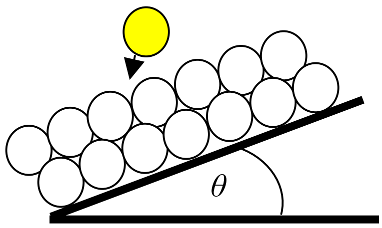


Figure 4.3



Two layers of identical spheres are arranged so that each sphere forming the upper layer sits on top of four other identical touching spheres arranged with their centres at the corners of a square, Figure 4.2. Calculate the separation of the planes through the centres of the spheres, h, in terms of the diameter of the spheres, d.

(e)

In practice it is found that piles of rough objects are conical and the angle that the face of the cone makes with the horizontal is about 33°.

Consider a simple two layer model as shown in (d). A slope, (angle θ with the horizontal), is constructed of two layers of spherical particles, Figure 4.3. A spherical particle is displaced and lands on the slope such that it drops symmetrically between four other particles, as in Figure 4.3, at a speed v. On impact it loses 50% of its KE. Calculate the smallest value of v so that it continues to bounce down the slope. What criticism can be made of this model and its ability to predict the behaviour of non-attracting particles?

(f) Clay slopes are never stable. Suggest a reason for this. [20]