BAAO
British Astronomy and Astrophysics Olympiad

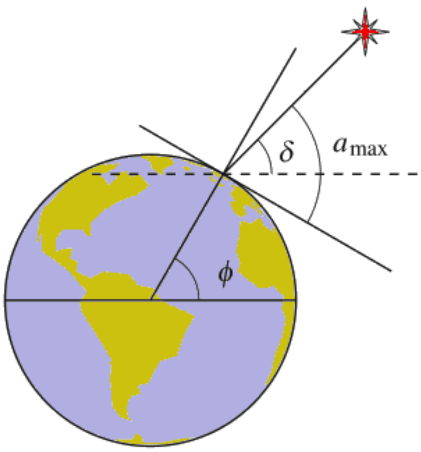
Astronomy & Astrophysics Challenge

September - December 2022

Solutions and marking guidelines

- The total mark for each question is in **bold** on the right-hand side of the table. The breakdown of the mark is below it.
- There is an explanation for each correct answer for the multiple-choice questions. However, the students are only required to write the letter corresponding to the right answer.
- In Section C, students should attempt **either** Qu 13 **or** Qu 14. If both are attempted, consider the question with the higher mark.
- Answers to two or three significant figures are generally acceptable. The solution may give more than that, especially for intermediate stages, to make the calculation clear.
- There are multiple ways to solve some of the questions; please accept all good solutions that arrive at the correct answer. Students getting the answer in a box will get all the marks available for that calculation / part of the question (students may not explicitly calculate the intermediate stages, and should not be penalised for this so long as their argument is clear)

Question	Answer	Mark
Section A		**10**
1.	A The other objects in the list of options were also first images released from JWST. Stephan's Quintet is a collection of galaxies, the Southern Ring Nebula shows the whole ring and is a planetary nebula around a dying star, whilst WASP-96 b is an exoplanet for which JWST was able to get spectra of its atmosphere as it transited its star.	1
2.	A Named after the sister of Apollo, it is the new programme to try and get people back to the Moon, including the first woman and person of colour. It will use the Space Launch System (SLS) rocket and the Orion spacecraft will carry the astronauts from Earth to lunar orbit. Artemis I (uncrewed) will hopefully launch in Nov 2022, with Artemis II (and its crew) in 2024.	1
3.	D Magnification for a single lens is equal to the ratio of image and object distance. For a two-lens system, it is the ratio of their focal lengths: $$\text{Magnification} = \frac{f_{\text{telescope}}}{f_{\text{eyepiece}}}$$ ∴ the 10 mm eyepiece will give the greatest magnification (x75)	1

4.	C The angular surface area of the sky is: $4\pi(1\ rad)^2 = 4\pi\left(\frac{180°}{\pi}\right)^2 = 41253 \text{ degrees}^2$ Assuming each object is in a square, the separation between objects is the length of one side of the square, giving $l = \sqrt{\frac{41250}{1.9 \times 10^6}} = 0.147° \sim 0.1°$ [Assuming each object is in a circle gives $l = 0.083° \sim 0.1°$ too]	1
5.	B If Jupiter is in opposition, that means the Sun is on the opposite side of the sky. Since it's happening a few days after the autumnal equinox that means Jupiter is where the Sun was a few days after the vernal equinox – that is Pisces (precession of the Earth's rotational axis means it is no longer in Aries, even though we still refer to it as the First Point in Aries!).	1
6.	D Since the Moon has moved a little further in its orbit around the Earth (which is in the same direction as the rotation), the Earth has to turn a little more each night for it to rise, and so it gets later from one night to the next. As viewed from the equator and ignoring eccentricity and inclination, the extra angle is $\frac{360°}{27.3} = 13.2°$ Over the course of a day the earth rotates 360°, so the angular speed of the sky is $\omega_{sky} = \frac{360°}{24 \text{ hr}} = 15° \text{ hr}^{-1}$ Consequently, the time needed to cover that extra angle is $t = \frac{13.2}{15} = 0.88 \text{ hr} = 52.7 \text{ min} \quad \therefore \text{time of next moonrise} = 22{:}53$ [Including the eccentricity and inclination of the Moon, the real time of moonrise varies from night to night by between 30 and 90 mins]	1
7.	C Given a latitude on Earth ϕ and stellar declination δ, the diagram is:  $\therefore a_{max} = 90° - \phi + \delta$ Since the latitude of Oxford is fixed, the star with the largest declination will culminate highest, and so it is Capella. [The longitude and right ascension simply affect the time it culminates, not how high it will be]	1

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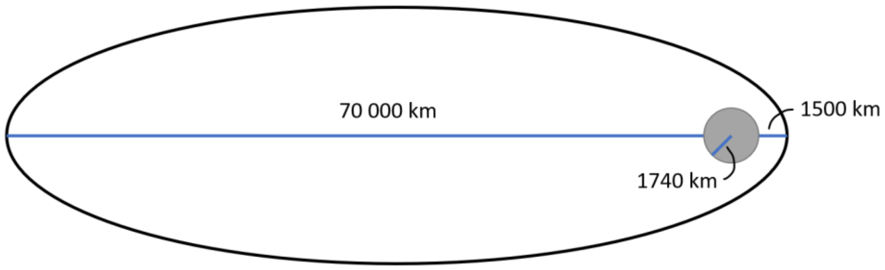

Semi-major axis, $a = \frac{1}{2}(70000 + 1500 + 2 \times 1740) = 37490 \text{ km}$

Using the formula given on page 2 for the periapsis:

$$r_{peri} = a(1-e) \therefore e = 1 - \frac{r_{peri}}{a} = 1 - \frac{1500 + 1740}{37490} = 0.91$$

(Students get the same answer if they use the apoapsis instead)
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Since the time interval is less than a year and we are told they both orbit in the same direction, the asteroid must be within the Earth's orbit. During the 300 days, the asteroid has completed more than one orbit as the Earth has moved further around the Sun in that time.

The total number of orbits by the asteroid in 300 days is $\left(1 + \frac{300}{365}\right)$, so the orbital period is $\frac{300}{1+\frac{300}{365}} = 165$ days.

Using Kepler's third law:

$$a = \sqrt[3]{\frac{GM_\odot}{4\pi^2}T^2} = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{4\pi^2} \times (165 \times 24 \times 3600)^2}$$

$$= 8.796 \times 10^{10} \text{ m } (= 0.586 \text{ au})$$

$$\therefore v = \sqrt{\frac{GM_\odot}{a}} = \sqrt{\frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{8.796 \times 10^{10}}} = 38.8 \text{ km s}^{-1}$$

[If the asteroid had been outside the orbit of the Earth, the only way to get the time interval less than a year would be if the asteroid orbited in the other direction to the Earth – this assumption leads to a solution with v = 18 km s^{-1}, which was not one of the options]
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Following an inverse square law, the brightness of Jupiter as viewed from Earth is

$$b \propto \frac{1}{d^2_{Sun \to Jupiter}} \frac{1}{d^2_{Earth \to Jupiter}}$$

Comparing the faintest to the brightest then gives

$$\frac{b_{faint}}{b_{bright}} = \frac{\frac{1}{\left(a(1+e)\right)^2} \frac{1}{(a(1+e)-1)^2}}{\frac{1}{\left(a(1-e)\right)^2} \frac{1}{(a(1-e)-1)^2}}$$

$$= \frac{\left(5.2(1-0.0489)\right)^2 (5.2(1-0.0489)-1)^2}{\left(5.2(1+0.0489)\right)^2 (5.2(1+0.0489)-1)^2} = 0.645$$

Converting it into a different in magnitudes using the given formula:

$$\Delta m = -2.5 \log\left(\frac{b_{faint}}{b_{bright}}\right) = -2.5 \log 0.645 = 0.476$$
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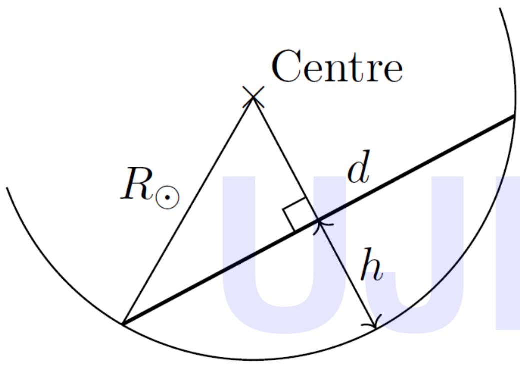
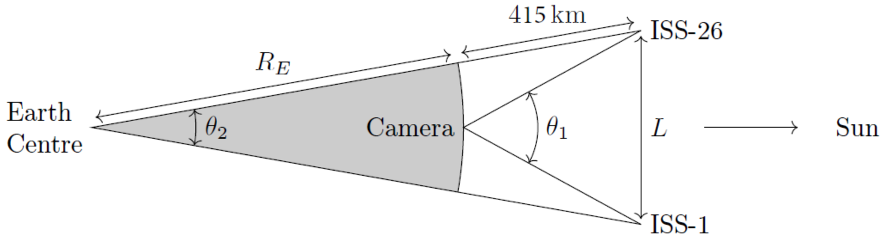
Section B		**10**
11.	a) Identifying that since Galatea's orbit is slightly smaller than the ring, the meaning of the orbital resonance is that for every 43 times Galatea orbits Neptune, a particle in the ring orbits 42 times $$\therefore T_{Galatea} = \frac{42}{43} T_{ring}$$ The mass of Neptune is not given, but it is not needed as both orbit in the same system and so Kepler's 3rd Law becomes $$\left(\frac{T_{Galatea}}{T_{ring}}\right)^2 = \left(\frac{a_{Galatea}}{a_{ring}}\right)^3$$ $$\therefore a_{ring} = a_{Galatea}\left(\frac{T_{ring}}{T_{Galatea}}\right)^{\frac{2}{3}} = 61953\left(\frac{43}{42}\right)^{\frac{2}{3}} = \boxed{62933\ \text{km}}$$ [Misinterpreting which way round the 42 and 43 should go leads to $a_{ring} = 60989$ km, and a max of 2 marks]	**[3]** 1 1 1
	b) Closest distance between the centre of the moon and a particle in the centre of the ring is the difference in their semi-major axes: $$d_{close} = a_{ring} - a_{Galatea} = 62933 - 61953 = 980\ \text{km}$$ Considering the formula for gravitational field strength $$g = \frac{GM}{d_{close}^2} \therefore M = \frac{g d_{close}^2}{G} = \frac{0.15 \times 10^{-3} \times (980 \times 10^3)^2}{6.67 \times 10^{-11}}$$ $$= \boxed{2.2 \times 10^{18}\ \text{kg}}$$ [Allow ecf from part a when working out d_{close}] The amplitude of the ripples is about 30 km and there are 42 of them in the ring, indicating that they must be due to Galatea – this is how the mass of Galatea was originally measured.	**[2]** 1 1
12.	a) Measure at least two observed wavelengths of hydrogen (allow ± 0.01 μm with the values below) Identify that the longest H wavelength corresponds to the Balmer-β line with rest frame wavelength of 486 nm Find that $z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} \approx \boxed{8.5}$ e.g. of measured wavelengths and values of redshift $4.62\ \mu\text{m} \rightarrow 486\ \text{nm} \therefore z = 8.51$ $4.12\ \mu\text{m} \rightarrow 434\ \text{nm} \therefore z = 8.49$ $3.90\ \mu\text{m} \rightarrow 410\ \text{nm} \therefore z = 8.51$	**[3]** 1 1 1

	b) Reading from the graph: $v \approx 2.1c$ [allow $\pm 0.1c$]	**[2]** 1
	Converting the speed to km s^{-1}, $d = \frac{v}{H_0} = \frac{2.1 \times 3.00 \times 10^5}{70} = \boxed{9000\text{ Mpc}}$	1
	[Allow full ecf for this part based upon their value for z in a. Final answer must be in Mpc for the final mark]	
Section C		**10**
13.	a) You can use Pythagoras to calculate the radius of the Sun, $R_\odot$, in terms of the length of the transit chord d, and the length of the perpendicular bisector h, as shown below.	**[4]**
	 $R_\odot^2 = \left(\frac{d}{2}\right)^2 + \left(R_\odot - h\right)^2$ $\therefore R_\odot = \frac{h}{2} + \frac{d^2}{8h}$	1
	We also measure the distance $d_{transit}$ between the outermost images of the ISS, so then $\theta_1 = d_{transit} \times \frac{31'29''}{2 \times R_\odot}$	1
	Measuring from the printout: $d = 12.4\text{ cm}, h = 3.8\text{ cm}, d_{transit} = 12.0\text{ cm}$ $\therefore R_\odot = \frac{3.8}{2} + \frac{12.4^2}{8 \times 3.8} = 6.96\text{ cm}$	1
	$\therefore \theta_1 = 12.0 \times \frac{31'29''}{2 \times 6.96} = \boxed{27'08''}$ $(= 27.1' = 0.452°)$	1
	[First mark is for any valid method to work out the radius of the Sun using the length of the chord – using the perpendicular bisector is the obvious route although any valid method is acceptable. Second mark is for a formula that relates the chord length to the angle subtended – only accept those that link it to the actual distance between first and last photo, rather than the whole chord. Third mark is for a sensible distance for the radius of the Sun (allowing for different scales on the printout). Acceptable range for the fourth mark is $\pm 1'$]	

b) **[3]**

Since we assume the transit took place whilst the Sun was at the zenith, we can draw the following diagram



1

From simple geometry (because the angles θ_1 and θ_2 are small) we have that

$$L \approx \frac{\theta_1}{360°} \times 2\pi \times 415 \text{ km} \quad \text{and} \quad L \approx \frac{\theta_2}{360°} \times 2\pi \times (R_E + 400) \text{ km}$$

1

and therefore

$$\theta_2 \approx \theta_1 \times \frac{415}{415 + R_E}$$

$$= 27'08'' \times \frac{415}{415 + 6370} = \boxed{1'40''} \quad (= 1.66' = 0.0277°)$$

1

[First mark is for a sensible diagram relating θ_1 and θ_2, whilst second mark is for a sensible method to relate them in an algebraic expression. Allow consistent value of θ_2 from their value of θ_1 for third mark (expect $\pm 0.06'$ but allow ecf). It is possible to derive an analytical expression for θ_2 without using the approximations by using the sine rule on three separate triangles. This obtains (in radians)

$$\theta_2 = \theta_1 - 2\sin^{-1}\left[\frac{R_E}{R_E + 415}\sin\left(\frac{\theta_1}{2}\right)\right] = 4.83 \times 10^{-4} \text{ rad} = 1'40''$$

This approach gains all of the marks]

c) **[3]**

The time between the first and last photographs to is calculated to be

$$T = 93 \text{ min} \times \frac{\theta_2}{360°} = 93 \text{ min} \times \frac{1'40''}{360°} = 0.429 \text{ s}$$

1

Hence the time interval between photographs is

$$\Delta T = \frac{T}{25} = \frac{0.429}{25} = 0.0172\ s$$

1

Finally, the frame rate is

$$\text{Frame rate} = \frac{1}{\Delta T} = \frac{1}{0.0172} = \boxed{58.3 \text{ fps}}$$

1

[Accept correct propagation of their value of θ_2 (expect ± 2 fps but allow ecf). Second mark requires dividing by 25 (number of intervals) **not** 26 (number of photographs)]

	According to the photographer, the actual frame rate of the camera was 80 fps. The difference can be accounted for through a much more complicated calculation involving the latitude of the observer, the altitude and azimuth coordinates of the Sun, and the true orbital parameters of the ISS.	
14.	a) Calculating the intensity at Earth from the Sun $$b_\odot = \frac{L_\odot}{4\pi(1\text{ au})^2} = \frac{3.85\times10^{26}}{4\pi(1.50\times10^{11})^2} = 1361.7\text{ W m}^{-2}$$ Since we know the apparent magnitude of this intensity, we can work it out for an apparent magnitude of zero $$\frac{b_\odot}{b_0} = 10^{-0.4(m_\odot - 0)}$$ $$\therefore b_0 = \frac{b_\odot}{10^{-0.4 m_\odot}} = \frac{1361.7}{10^{-0.4\times-26.832}} = 2.52\times10^{-8}\text{ W m}^{-2}$$ If the JWST is a distance d_{JWST} from the Earth, then the intensity at JWST from the Sun is $$b_{\odot\to JWST} = \frac{L_\odot}{4\pi\left(1\text{ au} + \mathrm{d_{JWST}}\right)^2}$$ $$= \frac{3.85\times10^{26}}{4\pi(1.50\times10^{11} + 1.5\times10^{9})^2} = 1334.8\text{ W m}^{-2}$$ Reflected power is $$L_{JWST} = 0.9\times b_{\odot\to JWST}\times(14\times21) = 353\text{ kW}$$ Brightness of JWST on Earth $$b_{JWST} = \frac{L_{JWST}}{4\pi d_{JWST}^2} = \frac{353196}{4\pi(1.5\times10^{9})^2} = 1.25\times10^{-14}\text{ W m}^{-2}$$ Comparing this to the intensity for an apparent magnitude of zero, $$m_{JWST} = -2.5\log\left(\frac{b_{JWST}}{b_0}\right) = -2.5\log\left(\frac{1.25\times10^{-14}}{2.52\times10^{-8}}\right) = \boxed{15.76}$$ [This is only one possible method – be generous to credit all other valid approaches. Some students may not evaluate any intermediate steps and keep things algebraic, resulting in the expression: $$m_{JWST} = m_\odot + 2.5\log\left(\frac{\frac{1}{4\pi(1\text{ au})^2}}{0.9\times\frac{14\times21}{4\pi\left(1\text{ au}+d_{JWST}\right)^2}\times\frac{1}{4\pi d_{JWST}^2}}\right)$$ This approach receives full marks]	**[7]** 1 1 1 1 1 2

	In practice, JWST is in a halo orbit around L_2 and so there are some extra geometric effects that mean it is typically around +16 to +17 as viewed from most latitudes.	
	b) Comparing the received intensity of the JWST to a +6 star: $$\frac{b_6}{b_{JWST}} = 10^{-0.4(6-15.76)} = 8029$$ So this means the area of telescope aperture needs to be 8029 times larger than the fully night-adapted eye so that it appears equally bright (the same number of received photons per second) $$\therefore \left(\frac{D_{tele}}{6\text{ mm}}\right)^2 = 8029$$ $$\therefore D_{tele} = \boxed{54\text{ cm}}$$ This is rather large and so corresponds to the telescopes found in fixed observatories, although smaller diameter telescopes can be used to image it by doing a long exposure of the sky. The image from the Virtual Telescope Project displayed in Figure 5 in the paper is taken by a 43.2 cm diameter telescope with a 300 second exposure time.	**[3]** 1 1 1