Astro Round 1 Mock Paper Solutions

Note for markers:

- Answers to two or three significant figures are generally acceptable. The solution may give more in order to make the calculation clear. Units should be present on final answers when appropriate.
- Allow ecf with penalty only applied at each distinct mistake.
- There are multiple ways to solve some of the questions; please accept all good solutions that arrive at the correct answer
- Students getting the answer in a $\boxed{\text{box}}$ will get all the marks available for that calculation / part of the question (as indicated in red), so long as there are no unphysical / nonsensical steps or assumptions made (students may not explicitly calculate the intermediate stages and should not be penalised for this so long as their argument is clear).

Section 1 Mark Scheme – Q1 [Short Questions]

A.

If $T_{Europa} = 2 \times T_{Io} \Rightarrow \omega_{Europa} = \frac{1}{2}\omega_{Io}$ [1]

Given $\omega^2 r^3 = \text{constant} \therefore \omega_{Europa}^2 r_{Europa}^3 = \omega_{Io}^2 r_{Io}^3$

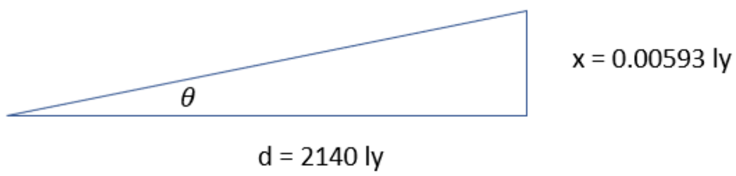
$\therefore r_{Europa} = r_{Io}\left(\frac{\omega_{Io}}{\omega_{Europa}}\right)^{2/3} = r_{Io} \times 2^{2/3}$ [1]

From circular motion, $a = \omega^2 r$

$\therefore \frac{a_{Io}}{a_{Europa}} = \left(\frac{\omega_{Io}}{\omega_{Europa}}\right)^2 \frac{r_{Io}}{r_{Europa}} = 2^2 \times \frac{1}{2^{2/3}} = \boxed{2.52}$ [1] [3]

B.

No need to convert units as they will cancel out when working out the angular separation



Using the small angle approximation that $\tan\theta \approx \theta$

$\therefore \theta \approx \frac{x}{d} = \frac{0.00593}{2140} = 2.77 \times 10^{-6}$ radians [1]

For a circular aperture, the minimum angle for resolving is $\theta = \frac{1.22\lambda}{D}$

$\therefore D = \frac{1.22\lambda}{\theta} = \frac{1.22\times550\times10^{-9}}{2.77\times10^{-4}} = \boxed{0.242\text{ m}}$ [1] [2]

C.

Given $L \propto R^2 T^4 \therefore \frac{L_{new}}{L_{old}} = \left(\frac{R_{new}}{R_{old}}\right)^2 \left(\frac{T_{new}}{T_{old}}\right)^4 \therefore \frac{T_{new}}{T_{old}} = \sqrt[4]{\frac{4000}{200^2}} = 0.562$ [1]

Given $\lambda \propto \frac{1}{T} \therefore \lambda_{new} = \lambda_{old} \times \frac{T_{old}}{T_{new}} = 500 \times \frac{1}{0.562} = \boxed{889\ \mathrm{nm}}$ [1] [2]

D.

$E = \frac{hc}{\lambda} \therefore \lambda_0 = \frac{hc}{E} = \frac{6.63\times10^{-34}\times3.00\times10^{8}}{2.55\times1.60\times10^{-19}} = 487.5\ \mathrm{nm}$ [1]

We can now work out the redshift: $z = \frac{\Delta\lambda}{\lambda_0} = \frac{5.4}{487.5} = 0.0111$ [1]

From this we can get the recessional speed: $z = \frac{v}{c} \therefore v = zc = 3323\ \mathrm{km\ s^{-1}}$ [1]

Finally, we can use Hubble's Law to get the distance: $d = \frac{v}{H_0} = \frac{3323}{70} = \boxed{47.5\ \mathrm{Mpc}}$ [1] [4]

E.

Since the time interval is less than a year and we are told they both orbit in the opposite direction, the asteroid must be outside the Earth's orbit. During the 300 days, the asteroid has completed less than one orbit as the Earth has moved further around the Sun in that time.

The total number of orbits by the asteroid in 300 days is $\left(1 - \frac{300}{365}\right)$,

so the orbital period is $\frac{300}{1-\frac{300}{365}} = 1684.6\ \mathrm{days}$ [1]

Using Kepler's third law: $a = \sqrt[3]{\frac{GM_\odot}{4\pi^2}T^2} = \sqrt[3]{\frac{6.67\times10^{-11}\times1.99\times10^{30}}{4\pi^2} \times (1684.6 \times 24 \times 3600)^2}$

$= 4.145 \times 10^{11}\ \mathrm{m}\ (= 2.76\ \mathrm{au})$ [1]

$\therefore v = \sqrt{\frac{GM_\odot}{a}} = \sqrt{\frac{6.67\times10^{-11}\times1.99\times10^{30}}{4.145\times10^{11}}} = \boxed{17.9\ \mathrm{km\ s^{-1}}}$ [1] [3]

F.

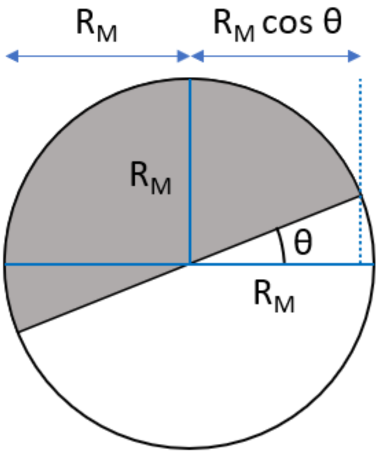
Nuclear density: $\rho = \frac{m_p}{\frac{4}{3}\pi r_p^3} = \frac{1.67\times10^{-27}}{\frac{4}{3}\pi(10^{-15})^3} = 3.99 \times 10^{17}\ \mathrm{kg\ m^{-3}}$ [1]

Volume of neutron star: $V = \frac{M}{\rho} = \frac{1.4\times1.99\times10^{30}}{3.99\times10^{17}} = 6.99 \times 10^{12}\ \mathrm{m^3}$ [1]

Radius of neutron star: $R = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3\times6.99\times10^{12}}{4\pi}} = \boxed{11.9\ \mathrm{km}}$ [1] [3]

G.

Suitable diagram of the situation (here it is viewed from above the lunar North pole) [1]



The shadow fraction is the ratio of the area in shadow over the whole area of the lunar disc, however since the vertical diameter is the same for both, it is only the ratio of horizontal diameters that matters

Shadow fraction $= \frac{R_M + R_M\cos\theta}{2R_M} = \frac{1}{2}(1+\cos\theta)$ [1]

If 0.6% is illuminated, 99.4% is in shadow

$\therefore 0.994 = \frac{1}{2}(1+\cos\theta)$

$\therefore \cos\theta = 0.988 \therefore \theta = 0.155 \text{ rad} \ (= 8.89°)$ [1]

Assuming it has a circular orbit then the angular velocity will be constant:

$\therefore \theta = \omega t = \frac{2\pi t}{T}$ [1]

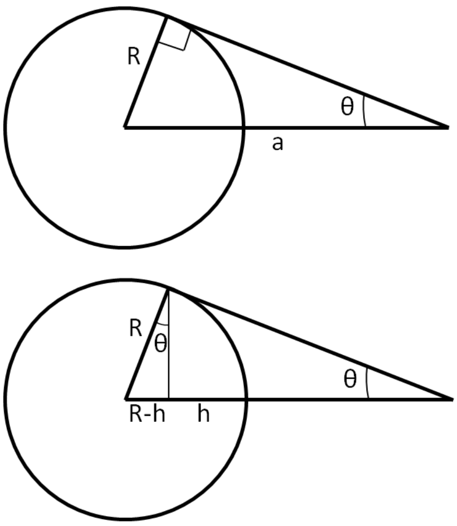
where θ is in radians, T is the lunar period and t is the time since the last new moon

$\therefore t = \frac{\theta T}{2\pi} = \frac{0.155 \times 29.53}{2\pi} = 0.729 \text{ days} = \boxed{17.5 \text{ hours}}$ [1] [5]

[Allow formula for θ for fourth mark in degrees. Must be in hours for the final mark. Some students might work out the shadow fraction by having the ratio of (semicircle + half ellipse) / circle, giving $\frac{\frac{1}{2}\pi R_M^2 + \frac{1}{2}\pi R_M \cdot R_M\cos\theta}{\pi R_M^2}$ which cancels down to the same expression – note that the area of an ellipse was given on the formula sheet]

H.

Radius of orbit from Kepler's 3rd Law: $a = \sqrt[3]{\frac{GM_\oplus T^2}{4\pi^2}} = \sqrt[3]{\frac{6.67\times10^{-11}\times5.97\times10^{24}\times(\frac{1}{2}\times24\times3600)^2}{4\pi^2}}$



$= 2.66 \times 10^7 \text{ m}$ [1]

$\theta = \sin^{-1}\left(\frac{R_\oplus}{a}\right) = \sin^{-1}\left(\frac{6.37\times10^6}{2.66\times10^7}\right) = 13.9°$ [1]

The area of a 'zone' of a sphere is $2\pi Rh$ where h is the radial height of the zone. From the geometry of the situation:

$\sin\theta = \frac{R-h}{R} \quad \therefore h = R(1-\sin\theta)$ [1]

$= 0.76\, R_\oplus$ [1]

Fraction of surface area:

$\frac{\text{Area of zone}}{\text{Surface area of Earth}} = \frac{2\pi R_\oplus(0.76R_\oplus)}{4\pi R_\oplus^2} = \boxed{38.0\%}$ [1] [5]

I.

We can calculate the apparent magnitude of the Sun as viewed from Earth

$$m_\odot = \mathcal{M}_\odot + 5\log\left(\frac{d}{10}\right) = 4.74 + 5\log\left(\frac{1.50\times10^{11}}{10\times3.09\times10^{16}}\right) = -26.83 \qquad [1]$$

Calculating the intensity at Earth from the Sun

$$b_\odot = \frac{L_\odot}{4\pi(1\text{ au})^2} = \frac{3.83\times10^{26}}{4\pi(1.50\times10^{11})^2} = 1354.6\text{ W m}^{-2} \qquad [1]$$

Since we know the apparent magnitude of this intensity, we can work out the intensity for an apparent magnitude of zero

$$\frac{b_\odot}{b_0} = 10^{-0.4(m_\odot - 0)} \therefore b_0 = \frac{b_\odot}{10^{-0.4m_\odot}} = \frac{1354.6}{10^{-0.4\times-26.83}} = 2.51\times10^{-8}\text{ W m}^{-2} \qquad [1]$$

If the JWST is a distance d_{JWST} from the Earth, then the intensity at JWST from the Sun is

$$b_{\odot\to JWST} = \frac{L_\odot}{4\pi(1\text{ au}+d_{JWST})^2} = \frac{3.83\times10^{26}}{4\pi(1.50\times10^{11}+1.5\times10^{9})^2} = 1327.9\text{ W m}^{-2} \qquad [1]$$

Reflected power is

$$L_{JWST} = 0.9\times b_{\odot\to JWST}\times(14\times21) = 351\text{ kW} \qquad [1]$$

Brightness of JWST on Earth

$$b_{JWST} = \frac{L_{JWST}}{4\pi d_{JWST}^2} = \frac{351360}{4\pi(1.5\times10^{9})^2} = 1.24\times10^{-14}\text{ W m}^{-2} \qquad [1]$$

Comparing this to the intensity for an apparent magnitude of zero,

$$m_{JWST} = -2.5\log\left(\frac{b_{JWST}}{b_0}\right) = -2.5\log\left(\frac{1.24\times10^{-14}}{2.51\times10^{-8}}\right) = \boxed{15.76} \qquad [1] \qquad [7]$$

[This is only one possible method – be generous to credit all other valid approaches. Some students may not evaluate any intermediate steps and keep things algebraic, resulting in the expression:

$$m_{JWST} = \mathcal{M}_\odot + 5\log\left(\frac{1\text{ au in pc}}{10}\right) + 2.5\log\left(\frac{\frac{1}{4\pi(1\text{ au})^2}}{0.9\times\frac{14\times21}{4\pi(1\text{ au}+d_{JWST})^2}\times\frac{1}{4\pi d_{JWST}^2}}\right)$$

This approach receives full marks]

J. i)

First, we calculate the mass difference between four hydrogen nuclei and one helium nucleus:

$\Delta m = 4m_{\mathrm{H}} - m_{\mathrm{He}} = (4 \times 1.674 \times 10^{-27}) - 6.649 \times 10^{-27} = 4.7 \times 10^{-29}\ \mathrm{kg}$ [1]

We can then work out the energy released per reaction

$E_{\mathrm{p-p}} = \Delta mc^2 = 4.7 \times 10^{-29} \times (3.00 \times 10^8)^2 = \boxed{4.23 \times 10^{-12}\ \mathrm{J}}$ [1] [2]

ii)

We can calculate the number of reactions per second, N:

$N = \frac{L_\odot}{E_{\mathrm{p-p}}} = \frac{3.85\times10^{26}}{4.23\times10^{-12}} = 9.10 \times 10^{37}$ [1]

Since there are four hydrogen nuclei used per reaction, the total number of hydrogen nuclei fusing per second is $\boxed{3.64 \times 10^{38}}$ [1] [2]

The percentage of hydrogen mass converted into energy is simply the change in mass in the p-p chain divided by the total mass of hydrogen used in the p-p chain:

$\frac{\Delta m}{4m_{\mathrm{H}}} = \frac{4.7\times10^{-29}}{4\times1.674\times10^{-27}} = \boxed{0.702\%}$ [1] [1]

iii)

Next, we can work out the total mass of hydrogen available for fusion:

$M_{\mathrm{H}} = 13\% \times 71\% \times M_\odot = 1.84 \times 10^{29}\ \mathrm{kg}$ [1]

By using our answer for the percentage mass change of the hydrogen, we can calculate how much mass will be converted into energy by the process:

$\Delta M_{\mathrm{H}} = 0.702\% \times 1.84 \times 10^{29} = 1.29 \times 10^{27}\ \mathrm{kg}$ [1]

This can be converted into the total energy released by the Sun over its lifetime:

$E_{\mathrm{tot}} = \Delta M_{\mathrm{H}} c^2 = 1.29 \times 10^{27} \times (3.00 \times 10^8)^2 = 1.16 \times 10^{44}\ \mathrm{J}$ [1]

Assuming a constant luminosity, we can then calculate the hydrogen burning lifetime of the Sun:

$t_\odot = \frac{E_{\mathrm{tot}}}{L_\odot} = \frac{1.16\times10^{44}}{3.85\times10^{26}} = 3.01 \times 10^{17}\ \mathrm{s} = \boxed{9.56 \times 10^9\ \mathrm{years}}$ [1] [4]

[Must be in years for the final mark]

Section 2 Mark Scheme – Q2 [DART Mission]

a) i)

T = 11 h 55 min = 11.92 hours = 42900 s

Using Kepler's 3rd law, $M_{sys} = \frac{4\pi^2}{GT^2}r^3 = \frac{4\pi^2}{6.67\times10^{-11}\times42900^2} \times 1200^3 = \boxed{5.56 \times 10^{11}\ \text{kg}}$ [1] [1]

a) ii)

Assuming same density

$M_{Dimo} = \left(\frac{164}{780}\right)^3 \times M_{sys} = \boxed{5.15 \times 10^9\ \text{kg}}$ [1] [1]

$M_{Didy} = M_{sys} - M_{Dimo} = \boxed{5.51 \times 10^{11}\ \text{kg}}$ [1] [1]

b) i)

Before the collision, we are in the rest frame of Dimorphos, so $p_{Dimo,orig} = 0$

$p_{probe} = 570 \times 6140 = 3.50 \times 10^6\ \text{kg m s}^{-1}$ [1]

Conserving momentum after the collision,

$p_{Dimo,new} = 3.50 \times 10^6\ \text{kg m s}^{-1}$ [1]

$\Delta v = v_{new} = \frac{p_{Dimo,new}}{M_{Dimo}} = \frac{3.50\times10^6}{5.15\times10^9} = \boxed{0.680\ \text{mm s}^{-1}}$ [1] [3]

b) ii)

Assuming a circular orbit:

$E_{tot} = GPE + KE = -\frac{GM_{Dimo}M_{Didy}}{r} + \frac{1}{2}M_{Dimo}\left(\frac{GM_{Didy}}{r}\right) = -\frac{GM_{Dimo}M_{Didy}}{2r}$ [1]

Before the collision:

$v_{orig} = \sqrt{\frac{GM_{Didy}}{r}} = \sqrt{\frac{6.67\times10^{11}\times5.51\times10^{11}}{1200}} = 0.174938\ \text{m s}^{-1}$ [1]

After the collision:

$v_{new} = v_{orig} - \Delta v = 0.174258\ \text{m s}^{-1}$ [1]

$E_{tot,new} = -\frac{GM_{Dimo}M_{Didy}}{r_{orig}} + \frac{1}{2}M_{Dimo}v_{new}^2$

$= -\frac{6.67\times10^{-11}\times5.15\times10^9\times5.51\times10^{11}}{1200} + \frac{1}{2} \times 5.15 \times 10^9 \times 0.174258^2$

$= -7.94 \times 10^7\ \text{J}$ [1]

$r_{new} = -\frac{GM_{Dimo}M_{Didy}}{2E_{tot,new}} = -\frac{6.67\times10^{-11}\times5.15\times10^9\times5.51\times10^{11}}{2\times-7.94\times10^7} = 1190.8\ \text{m}$ [1]

$\therefore \Delta r = 1200 - 1190.8 = \boxed{9.2\ \text{m}}$ [1] [6]

b) iii)

$$T_{new} = \sqrt{\frac{4\pi^2}{GM_{sys}} r_{new}^3} = \sqrt{\frac{4\pi^2}{6.67\times10^{-11}\times5.56\times10^{11}} \times 1190.8^3} = 42405 \text{ s}$$ [1]

$\therefore \Delta T = 495 \text{ s} = \boxed{8.24 \text{ min}}$ [1] [2]

(Watch out for those using M_{Didy} rather than M_{sys} in the final step, and hence get $T_{new} = 42603\ s$ and $\Delta T = 297 \text{ s} = 4.95 \text{ min}$ – they lose the final mark. If penalised here for it, do not penalise in c))

c)

$$T_{real} = 42900 - (32 \times 60) = 40980 \text{ s}$$

$$r_{real} = \sqrt[3]{\frac{GM_{sys}}{4\pi^2} T_{real}^2} = \sqrt[3]{\frac{6.67\times10^{-11}\times5.56\times10^{11}}{4\pi^2} \times 40980^2} = 1163.9 \text{ m}$$ [1]

$$E_{tot,real} = -\frac{GM_{Dimo}M_{Didy}}{2r_{real}} = -\frac{6.67\times10^{-11}\times5.15\times10^9\times5.51\times10^{11}}{2\times1163.9} = -8.12 \times 10^7 \text{ J}$$ [1]

$$KE_{real} = E_{tot,real} - GPE = -8.12 \times 10^7 + \frac{6.67\times10^{-11}\times5.15\times10^9\times5.51\times10^{11}}{2\times1200} = 7.63 \times 10^7 \text{ J}$$ [1]

$$v_{real} = \sqrt{\frac{2KE_{real}}{M_{Dimo}}} = \sqrt{\frac{2\times7.63\times10^7}{5.15\times10^9}} = 0.172205 \text{ m s}^{-1}$$ [1]

$\therefore \Delta v_{real} = v_{orig} - v_{real} = \boxed{2.733 \text{ mm s}^{-1}}$ [1] [5]

$\therefore \beta = \frac{\Delta v_{real}}{\Delta v} = \frac{2.733}{0.680} = \boxed{4.02}$ [1] [1]

(using M_{Didy} instead of M_{sys} in first step leads to $\beta = 4.44$)

Section 2 Mark Scheme – Q3 [Stellar Processes]

a) i)

From Wien's displacement law, $\lambda_{star} = \frac{2.90\times10^{-3}}{35800} = 81.0 \text{ nm}$ [1]

$E_\gamma = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3.00\times10^{8}}{81.0\times10^{-9}} = 2.46\times10^{-18}\text{ J} = \boxed{15.3 \text{ eV} > 13.6 \text{ eV}}$ ($\therefore$ ionising) [1] [2]

[For the second mark allow a comparison in J, or in terms of lambda (i.e $\lambda < 91.4$ nm)]

a) ii)

$N = \frac{L}{E_\gamma} = \frac{1.47\times10^{5}\times3.83\times10^{26}}{2.46\times10^{-18}} = 2.29\times10^{49} \text{ photons s}^{-1}$ [1]

$r_s = \left(\frac{3N}{4\pi\alpha}\right)^{1/3} n_H^{-2/3} = \left(\frac{3\times2.29\times10^{49}}{4\pi\times3.1\times10^{-19}}\right)^{1/3} (10^{9})^{-2/3} = \boxed{2.60\times10^{16}\text{ m}}$ [1] [2]

b)

$p_{rad} = \frac{L}{4\pi r^2 c} = \frac{1.47\times10^{5}\times3.85\times10^{26}}{4\pi\times(2.61\times10^{16})^2\times3.00\times10^{8}} = 2.20\times10^{-11}\text{ Pa}$ [1]

$M_{collapse} = \frac{1.18k_B^2T_{neb}^2}{p_0^{1/2}G^{3/2}m_H^2} = \frac{1.18\times(1.38\times10^{-23})^2\times50^2}{(2.21\times10^{-11})^{1/2}\times(6.67\times10^{-11})^{3/2}\times(1.67\times10^{-27})^2}$

$= 7.88\times10^{31}\text{ kg} = \boxed{39.6\, M_\odot}$ [1] [2]

c) i)

$\frac{\bar{m}}{m_H} = \frac{2}{1+3X+0.5Y} = \frac{2}{1+3\times0.35+0.5\times0.65} = 0.84$

$\therefore \bar{m} = 0.84m_H = 0.84\times1.67\times10^{-27} = 1.41\times10^{-27}\text{ kg}$ [1]

$p_{gas} = \frac{\rho_c k_B T_c}{\bar{m}} = \frac{1.53\times10^{5}\times1.38\times10^{-23}\times1.57\times10^{7}}{1.41\times10^{-27}} = 2.357\times10^{16}\text{ Pa}$ [1]

$p_{rad} = \frac{4\sigma}{3c}T_c^4 = \frac{4\times5.67\times10^{-8}}{3\times3.00\times10^{8}}(1.57\times10^{7})^4 = 1.531\times10^{13}\text{ Pa}$ [1]

$\therefore p_{tot} = p_{gas} + p_{rad} = 2.357\times10^{16} + 1.531\times10^{13} = \boxed{2.359\times10^{16}\text{ Pa}}$ [1] [4]

c) ii)

$\frac{p_{rad}}{p_{tot}} = 6.5\times10^{-4} = \boxed{0.065\%}$ [1] [1]

Radiation pressure is NOT a significant contribution to the outward pressure in a star like the Sun [1] [1]

[In the most massive stars where ρ_c and T_c are much larger then radiation pressure is very important and ultimately becomes strong enough to prevent stars above a certain mass limit from forming]

c) iii)

For constant density $\rho(r) = \rho$ and $M(r) = \frac{4}{3}\pi r^3 \rho$

$\therefore p_{Grav} = \int_0^{R_{star}} \frac{GM(r)\rho(r)}{r^2}\,\mathrm{dr} = \int_0^{R_{star}} \frac{G\times\frac{4}{3}\pi r^3\rho^2}{r^2}\,\mathrm{dr} = \int_0^{R_{star}} \frac{4}{3}\pi G\rho^2 r\,\mathrm{dr} = \frac{2}{3}\pi G\rho^2 R_{star}^2$ [1]

For the white dwarf, $\rho_{av} = \frac{M_{Sun}}{\frac{4}{3}\pi R_{Earth}^3} = \frac{1.99\times10^{30}}{\frac{4}{3}\pi(6.37\times10^6)^3} = 1.84 \times 10^9\ \mathrm{kg\ m^{-3}}$ [1]

$\therefore p_{Grav} = \frac{2}{3}\pi G\rho^2 R_{star}^2$

$= \frac{2}{3}\pi \times 6.67 \times 10^{-11} \times (1.84 \times 10^9)^2 \times (6.37 \times 10^6)^2 = \boxed{1.91 \times 10^{22}\ \mathrm{Pa}}$ [1] [3]

This is $\boxed{812\ 000\ p_{tot}}$. (Allow $\sim 10^6\ p_{tot}$) [1] [1]

[This huge gravitational pressure is balanced by an outward electron degeneracy pressure – a quantum mechanical effect – to keep the white dwarf stable]

d) i)

$E_{tot} = V \times \frac{4\sigma}{c} T_{mean}^4 = \frac{4}{3}\pi R_{Sun}^3 \times \frac{4\sigma}{c} T_{mean}^4$

$= \frac{4}{3}\pi \times (6.96 \times 10^8)^3 \times \frac{4\times5.67\times10^{-8}}{3.00\times10^8} \times (4.73 \times 10^6)^4 = 5.34 \times 10^{38}\ \mathrm{J}$ [1]

$t = \frac{E_{tot}}{L_{Sun}} = \frac{5.34\times10^{38}}{3.85\times10^{26}} = 1.39 \times 10^{12} = \boxed{44\ 000\ \text{years}}$ (must be in years) [1] [2]

d) ii)

Assuming the photon travels at c between absorption and emission:

$t = N \times t_{step}$ and $t_{step} = \frac{\ell}{c}$ $\therefore N = \frac{ct}{\ell}$

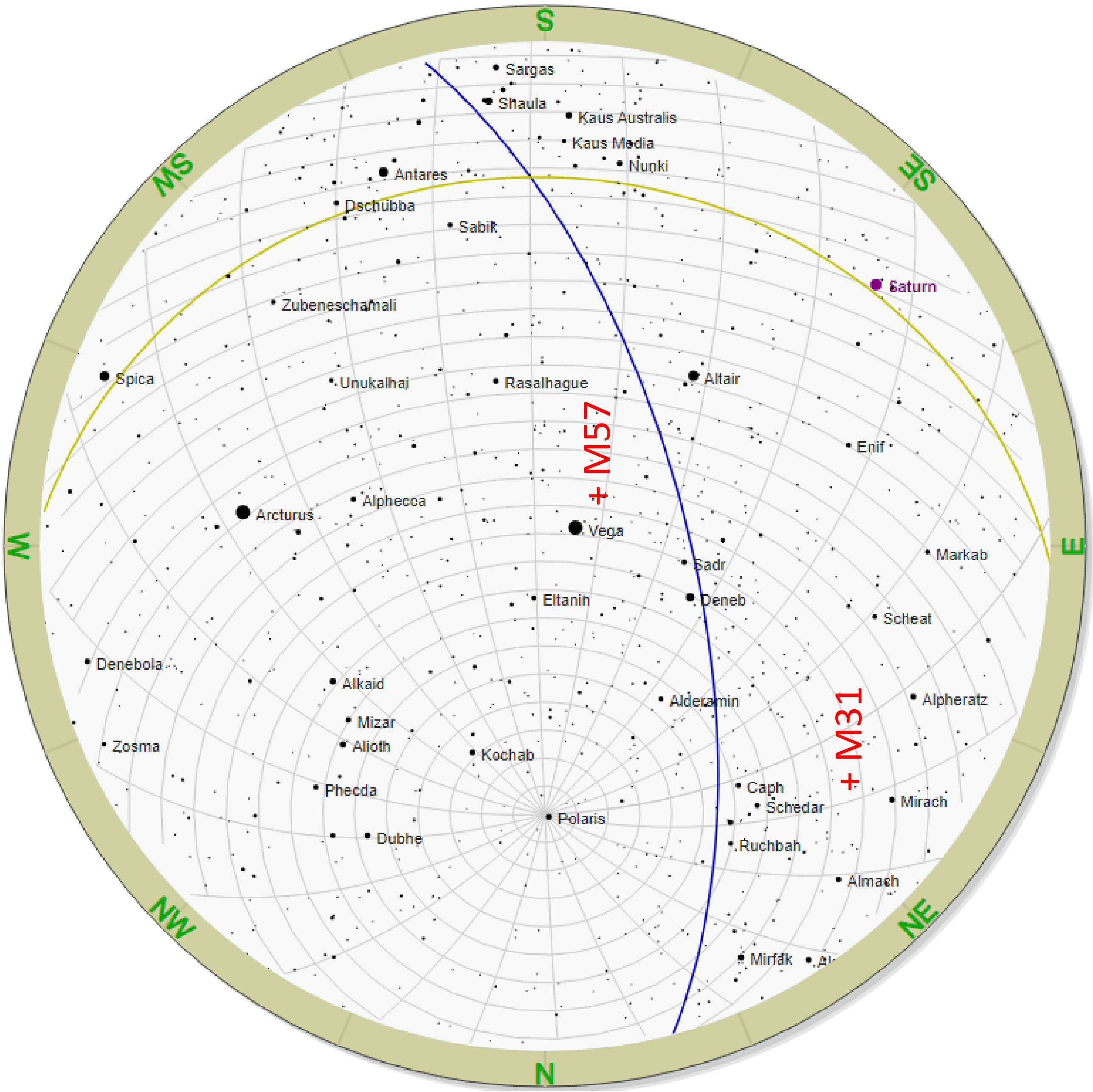
but $R_{Sun} = \ell\sqrt{N}$ $\therefore R_{Sun}^2 = \ell^2 N = \ell^2 \frac{ct}{\ell} = \ell ct$

$\therefore \ell = \frac{R_{Sun}^2}{ct} = \frac{(6.96\times10^8)^2}{3.00\times10^8\times1.39\times10^{12}} = \boxed{1.16\ \mathrm{mm}}$ [1] [1]

$N = \frac{R_{Sun}^2}{\ell^2} = \frac{(6.96\times10^8)^2}{(1.16\times10^{-3})^2} = \boxed{3.58 \times 10^{23}}$ [1] [1]

[A more careful calculation by Mitalas & Sills (1992) using a computer model that took into account the varying step length due to varying density found the diffusion time to be 170,000 years and the average step length to be 0.90 mm – not very far off these simple estimates.]

Section 2 Mark Scheme – Q4 [Observational Astronomy]



a) Spica		[1]	[1]
b) 42°	(accept 40° to 44°)	[2]	[2]
c) Arcturus	(α Boo)	[1]	[1]
Dubhe	(α UMa)	[1]	[1]
Vega	(α Lyr)	[1]	[1]

d) Any four from:

Virgo, Libra, Scorpius, Ophiuchus, Sagittarius, Capricornus, Aquarius, Pisces [4] [4]

(If more than four given, only mark the first four)

e) Capricornus (also accept Aquarius) [1] [1]

f) i) M31, M57 (if more than two given, only mark the first two) [2] [2]

ii) M57 [1] [1]

iii) (M31 is a galaxy)

Altitude = 20° (accept 15° to 25°) [1] [1]

Nearest compass direction = NE (North East) [1] [1]

g)

Time until Sun next crosses southern meridian = 14.5 hours [midday of next day – 21:30] [1]

Use of the known right ascension of a star to find vernal equinox will cross southern meridian in 6 h [1]

Sun is (14.5 – 6 =) 8.5 hours behind the vernal equinox [1]

Month is (8.5/24 × 12 =) 4.25 months after vernal equinox
so (late) July / (early) August (accept either) [1] [4]

[This is the sky above Kutaisi, Georgia, on 17th August 2022 – the day of the IOAA 2022 observational exam.]

Explained Answers for Q4

The first thing to do is the identify Polaris as the bright object on the meridian (the line from N to S), from here you can identify the location of Ursa Major ("the Plough"), which gives a way in to answering the first few questions and hence later ones.

a)

The handle of the Plough points to Arcturus ("arc to Arcturus") and on to Spica. Given the distance across the sky from the handle it must be Spica (we are told it is a star, rather than a bright planet)

b)

So long as altitude is linear with radial distance from the centre of the map, we can measure how high above the horizon Polaris is (approximately ¼ of the diameter along the meridian [representing 180°], hence approximately 45° - using a ruler on the page and measuring to the nearest mm gets 42°). Note the altitude of Polaris is your latitude (this can be realised with a minute's thought)

c)

Having identified Ursa Major we see that the upper of the two pointer stars to Polaris are missing, and that one is Dubhe (positioned to complete the quadrilateral, given the student has a good idea of its shape). We see that the handle points directly to Spica without a bright star intervening, so it must be that Arcturus is missing as well (approximately halfway between the end of the handle and Spica). Finally, we can make out Cygnus (the swan / Northern Cross – with the bright star Deneb) and Aquila (the eagle – with bright star Altair), two parts of the Summer Triangle, and so the third missing star must be the final vertex of that triangle – Vega (approximate position determined by knowing the Vega-Deneb line is roughly at 90° to the Vega-Altair line) d)

The constellations on the ecliptic (yellow line) will be the zodiacal ones (plus Ophiuchus). We know Spica (identified in a)) is in Virgo so that marks the Western end of the ecliptic in this image, so we just move through the zodiac in the direction of South (towards Sagittarius) and then East (finishing with Pisces). Most students would just mention Virgo (from Spica) and then the 3 next to it (Libra, Scorpius and Sagittarius [or Ophiuchus])

e)

Spica and Antares (in Scorpius) are near the ecliptic, so having identified them you have a good sense of the curve of the ecliptic. The planet would be next very bright object you encounter on the ecliptic (Capricornus and Aquarius don't have stars as bright as this object, which further gives it away)

f) i)

The question really tests a knowledge of the Messier numbers – to know what object is being referred to with its designation:

M1 is the Crab nebula in Taurus – this is one of zodiacal constellations not visible on the ecliptic, so must be **below** the horizon
M31 is the Andromeda galaxy – it is below the second "U" in the double-u of Cassiopeia, so it should be **visible**. Precise location is done by looking at the line of three bright stars in Andromeda (Almach, Mirach and Alpheratz and knowing it is on a line at 90° to that one starting from Mirach)
M42 is the Orion Nebula – as one of the most widely recognisable constellations in the sky, it is clear Orion is not in this sky map and so M42 must be **below** the horizon
M44 is the Beehive cluster in Cancer – another zodiacal constellation not visible on the ecliptic, so must be **below** the horizon
M45 is the Pleiades in Taurus – for the same reasons as M1 this must be **below** the horizon
M57 is the Ring Nebula in Lyra – this constellation contains Vega so we know it must be **visible**. It is approximately halfway between the bottom two stars in the quadrilateral of Lyra

f) ii)

The galactic equator (blue line) goes through the neck and body of Cygnus the swan, through Deneb. The galactic centre (on the galactic equator) is in the steam from the teapot Sagittarius. Also, it goes through Cassiopeia. With these pieces of knowledge, we can draw a reasonably accurate curve, and hence see that M57 is closer to it than M31

f) iii)

M31 is the galaxy, so we can work out it's altitude the same way we worked out the altitude of Polaris in b), and we can see the nearest compass direction from the ring on the outside of the image

g)

We're told the time is 21:30, and therefore we can estimate where the Sun is now (without knowing the time zone and longitude, we can't do this very accurately), namely, it will cross the southern meridian in roughly 14h30m (the time from now to midday the next day). We need to know additionally a coordinate of a star / point on the sky, and that the Sun was at the 'zero point' (the 'Vernal Equinox' in Pisces) on (roughly) 21st March. For example, the right ascension (RA) of Antares is about 16h30m (angular distance from the Vernal Equinox), and by eye, we estimate that around 1h30m ago, it would have been on the southern meridian. That is, stars on the southern meridian have an RA of 18h, and the Vernal Equinox will cross the meridian in 6 hours. The Sun is therefore around 8.5 hours (14h30m – 6h), or just over a third of a rotation (8.5/24 = 35.4%), behind the Vernal Equinox, where it was in March. The month is therefore just over one third of a year (35.4% of 12 = 4.25 months) after 21st March (approximately 2.75 months since 1st Jan) – this puts it into late July / early August (2.75 + 4.25 = 7.0 months since 1st Jan).

[Note 1: The real answer is mid-August in Georgia – we are off because Georgia is an hour ahead in local time (GMT+4) than you would expect from their longitude (about 45° so GMT+3). Correcting for this gives the Sun 9.5 hours behind the Vernal equinox and so gives a date of 7.5 months since 1st Jan]

[Note 2: Of course, the presence of the Summer Triangle high in the sky at this time of the evening might suggest that we were probably looking at June / July but this only applies near the Greenwich meridian – the route above is a more general way of working it out and conversely were you to know the month and GMT (from a sea-worthy clock) you could estimate your longitude, which is how sailors at sea could estimate where they were]