BPhO
British Physics Olympiad

BPhO Round 1

Section 2

15th November 2019

This question paper must not be taken out of the exam room

<u>Instructions</u>

Time: **5 minutes reading time (NO writing)** and then **1 hour 20 minutes** for writing.

Questions: Only **two questions** out of the five questions in *Section 2* should be attempted.

Each question contains independent parts so that later parts should be attempted even if earlier parts are incomplete.

Working: Working, calculations, explanations and diagrams, properly laid out, must be shown for full credit. The final answer alone is not sufficient. Writing must be clear. If derivations are required, they must be mathematically supported, with any approximations stated and justified.

Marks: Students are recommended to spend about 40 minutes on each question. Each question in *Section 2* is out of 25, with a **maximum of 50 marks from two questions** only.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used, but calculators cannot be programmable and must not have symbolic algebra capability.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. Graph paper and formula sheets should also be made available. Students should ensure that their **name** and their **school/college** are clearly written on each and every answer sheet. Number the pages.

Setting the paper: There are two options for sitting BPhO Round 1:

a. *Section 1* and *Section 2* may be sat in one session of 2 hours 40 minutes plus 5 minutes reading time (for *Section 2* only). *Section 1* should be collected in after 1 hour 20 minutes and then *Section 2* given out.

b. *Section 1* and *Section 2* may be sat in two sessions on separate occasions, with 1 hour 20 minutes plus 5 minutes reading time allocated for *Section 2*. If the paper is taken in two sessions on separate occasions, *Section 1* must be collected in after the first session and *Section 2* handed out at the beginning of the second session.

Important Constants

Constant	Symbol	Value
Speed of light in free space	c	$3{:}00 \quad 10^{8}$ m s ^{1}
Elementary charge	e	$1{:}60 \quad 10^{19}$ C
Planck's constant	h	$6{:}63 \quad 10^{34}$ J s
Mass of electron	m_{e}	$9{:}11 \quad 10^{31}$ kg
Mass of proton	m_{p}	$1{:}67 \quad 10^{27}$ kg
atomic mass unit	u	$1{:}661 \quad 10^{27}$ kg
(1 u is equivalent to 931:5 MeV=c^2)		
Gravitational constant	G	$6{:}67 \quad 10^{11}$ m^3 kg ^{1} s ^{2}
Acceleration of free fall at Earth's surface	g	$9{:}81$ m s ^{2}
Permittivity of free space	$''_0$	$8{:}85 \quad 10^{12}$ F m ^{1}
Permeability of free space	$_0$	$4 \quad 10^{7}$ H m ^{1}
Avogadro constant	N_{A}	$6{:}02 \quad 10^{23}$ mol ^{1}
Mass of Sun	M_{S}	$1{:}99 \quad 10^{30}$ kg
Radius of Earth	R_{E}	$6{:}37 \quad 10^{6}$ m

$$T_{(\mathrm{K})} = T_{(^\circ\mathrm{C})} + 273$$

$$\text{Volume of a sphere} = \tfrac{4}{3} \quad r^3$$

$$e^x \quad 1 + x + {:}{:}{:} \qquad x \quad 1$$

$$(1 + x)^n \quad 1 + nx \qquad x \quad 1$$

$$\frac{1}{(1+x)^n} \quad 1 \quad nx \qquad x \quad 1$$

$$\sin \qquad \text{for} \quad 1$$

$$\tan \qquad \text{for} \quad 1$$

$$\cos \quad 1 \quad {}^2{=}2 \qquad \text{for} \quad 1$$

Section 2

Question 2

This is about forces on dam walls and detecting cracks in them through wave reflection.

a) The dam shown in **Fig. 1** is used to generate hydroelectric power. The difference in height between the water levels of the dam is $61\,\mathrm{m}$, and the peak flow rate of the water through the dam is $3380\,\mathrm{m^3\,s^{-1}}$. Estimate the maximum power output of the hydroelectric facility. **(3)**

Figure 1: HEP dam.
credit: https://upload.wikimedia.org/wikipedia/commons/b/b2/Epa-archives_the_dalles_dam-cropped.jpg

b) In practice not all of the gravitational potential energy of the water behind the dam is transferred by the generators, and the water emerges from the dam with a velocity of $8.0\,\mathrm{m\,s^{-1}}$. Calculate the efficiency of the energy transfer process from the water. **(3)**

c) The dam can be considered to be a concrete prism of triangular cross-section, with height $h = 60\,\mathrm{m}$, a base of thickness $t = 60\,\mathrm{m}$, and a width of $w = 2700\,\mathrm{m}$. The cross-section of the dam is illustrated in **Figure 2**. Points **F** and **C** are at the bottom of the dam. The water level is at the top of the dam wall.

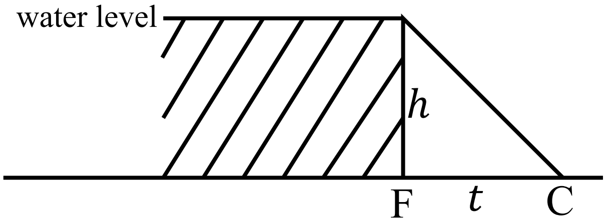


Figure 2

(i) Calculate the resultant force on the face of the dam due to water pressure in terms of h, w, ρ and g, where ρ is the density of water.

(ii) Calculate the resultant moment (torque) about **F** due to this force, and calculate the effective height at which the resultant force acts to produce this moment.

The centre of mass of a uniform triangle is at the point of intersection of its medians (the lines joining its vertices to the midpoints of the opposite sides). Thus the centre of mass lies two-thirds of the way along each median.

(iii) For the dam, state the position of the centre of mass of the dam relative to **F**.

(iv) Calculate the two turning moments acting on the dam about **C**, due to the water pressure and the weight of the dam.

(v) The frictional force of the dam on the ground beneath is given by $f = \quad N$ where N is the normal reaction force of the ground, and $\quad = 0{:}75$. Calculate the minimum density of concrete that could be used so that the dam will not slide over the ground.

(vi) Calculate the minimum density of concrete used to construct the dam so that it will not tip up about point **C**.
Density of water is $_w = 1000\,\mathrm{kg\,m}\quad{}^{3}$

(10)

d) There is a significant danger of cracks forming in dams, leading to catastrophic failure with little or no warning. One form of non-destructive testing uses ultrasonic waves to investigate the interior of the concrete.
When a wave strikes a boundary (interface) between two different media it has to obey two constraints.

the displacement of the material by the wave must be the same either side of the boundary.

the energy transferred into the boundary must be the same as the total energy reflected and transmitted.

If an incident wave has an amplitude A_i, the reflected wave amplitude A_r and the transmitted wave amplitude A_t, then we can write:

$$A_i + A_r = A_t \qquad (1)$$

The energy of a wave is proportional to the square of the amplitude, A^2, and the rate of energy flow in a medium is given by $\frac{E}{t} = !^2 Z A^2$, where Z stands for the *acoustic impedance* and is a parameter of the medium, and $!$ is a constant (it cancels in all our equations and can be ignored).

(i) From the energy flow through a boundary from medium 1 to medium 2, show that

$$Z_1 A_i^2 \quad Z_1 A_r^2 = Z_2 A_t^2 \qquad (2)$$

where Z_1 and Z_2 are the impedances of the media either side of the boundary, as illustrated in **Figure 3**.
Using these two requirements (eq. (1) and (2)) for a wave crossing an interface between two media at normal incidence:

(ii) show that $\frac{A_r}{A_i} = \frac{(Z_1 \quad Z_2)}{(Z_1 + Z_2)}$, and

(iii) derive an expression for $\frac{A_t}{A_i}$ in terms of Z_1 and Z_2.

(iv) If $Z_1 = Z_2$, what can be said about the transmitted and reflected waves and the values of $\frac{A_r}{A_i}$ and $\frac{A_t}{A_i}$?

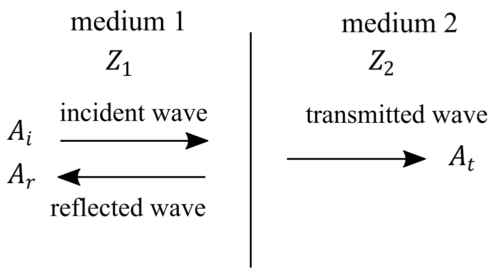


Figure 3

(v) When $Z_2 > Z_1$, what effect does this produce?

The energy reflected back can be used to charaterise cracks in the concrete. The *coefficient of reflection*, R, is the ratio of the reflected to the incident energy flow, such that $R = \frac{A_r^2}{A_i^2}$. The *coefficient of transmission*, T, is the ratio of the transmitted energy flow in medium 2 to the incident energy flow in medium 1.

(v) Obtain expressions for R and T in terms of Z_1 and Z_2.

(vi) Calculate a value for $R + T$.

For a medium, $Z = \ v_{sound}$, where is the density of the medium, and v_{sound} is the speed of sound in the medium. Values for the densities and sound speeds in air, concrete and water are given in **Table 1**.

medium	density kg m ^{3}	sound speed m s ^{1}
air	1.2	330
concrete	2400	3700
water	1000	1480

Table 1

(vii) Calculate the percentage of ultrasound energy reflected from a crack in the concrete when

i. the crack is filled with air,
ii. the crack is filled with water.

(9)

[25 marks]

Question 3

Several mechanical fairground rides are analysed in this question.

a) A particle of mass m moves at a constant rate in a horizontal circular path of radius r. State why there must be a resultant force acting on the particle; indicate its direction on a diagram, and state its magnitude both in terms of its speed v and its angular speed ω

(2)

b) A fairground ride consists of a set of chairs hanging from cables of length $\ell = 4.0\,\mathrm{m}$, which are attached to circular support of radius $a = 6.0\,\mathrm{m}$, as shown in **Figure 4** below. Each passenger sits in a light chair, with combined mass m, which can be treated as a point mass at the end of each cable. The ride rotates at angular speed ω, and the cable hangs at an angle of $\theta = 30^\circ$ to the vertical.
Obtain an expression for the angular velocity ω in terms of ℓ, a, θ and g and then evaluate ω for the given values.

(2)

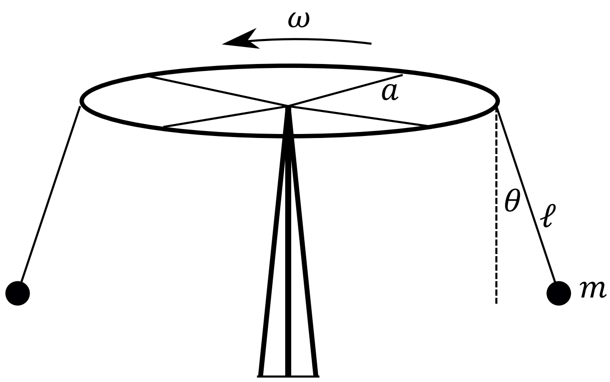
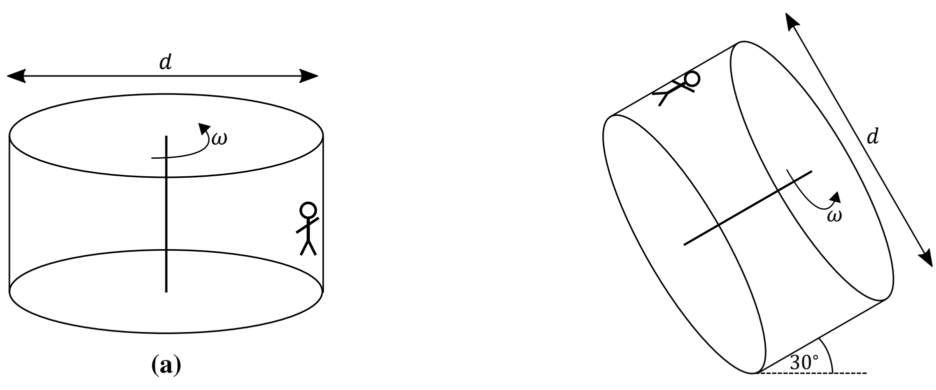


Figure 4

c) The "wall of death" ride consists of a $d = 10\,\mathrm{m}$ diameter drum which rotates at an angular rate ω, shown in **Figure 5a**. Passengers stand against the inside vertical, rough wall and as the ride speeds up to rate ω, they are held against the wall and cannot slide down due to friction with the wall.



(a) **(b)**

Figure 5

(i) Sketch a diagram of the three forces acting on a passenger who is held against the wall.

(ii) If the frictional force is given by $f = \mu N$ where N is the normal reaction force of the wall, and $\mu = 0.4$, calculate the minimum value of ω for the drum in order for passengers to be held against the wall.

(iii) The rotating drum is now tilted through 60° so that its axis of rotation is $\theta = 30^\circ$ above the horizontal (**Figure 5b**). Sketch a diagram of the three forces acting on a passenger when they are at the highest point of the circular path.

(iv) The frictional force f remains as μN with $\mu = 0.4$ although the value of f will be different as the wall is no longer vertical. Given this condition on f, calculate the new value of the minimum angular velocity, ω_t of the drum so that passengers are held against the wall at the highest point.

(6)

d) Roller coasters operate using gravity and are not powered. A roller coaster similar to that illustrated in **Fig. 6** can be modelled as a circular loop of radius of 20 m connected to a straight sloping track, shown in **Figure 7**.

Figure 6: credit: https://www.themeparkreview.com/parks/p_294_2875_gold_reef_city_golden_loop

(i) Now use the simplified model of **Figure 7**. Estimate the minimum height h of the top of the slope from which a car must start in order to remain in contact with the track at all times when the car travels around the loop of radius $r = 20$ m without falling. Friction can be neglected.
Hint: consider the forces acting on the car at the top of the loop, and the minimum speed needed to remain in contact.

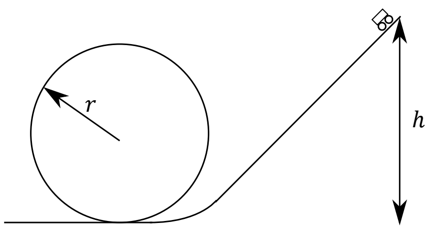


Figure 7

(ii) In this simplified model of the loop ride, the straight track joins on to the circular track at the bottom of the slope. Friction now acts on the carriage such that the frictional force is $f = \mu N$, where N is the normal contact force of the track on the carriage and $\mu = 0.05$.

i. Draw a force diagram for the resultant force acting on the carriage as it accelerates down the straight track.

ii. For a track length ` = 60 m at an angle = 40 above the horizontal, by what percentage does friction reduce the speed of the carriage at the bottom of the sloped track?

(iii) We again neglect friction. The straight track now joins a curved track with a radius of curvature of `=2. What would be the change in the normal contact force acting on a 60 kg passenger from his seat at the moment the carriage enters the curve?

(8)

e) A student has an even more unusual idea for a fun-fair ride. This can be modelled as two bars (**A** and **B**) each of mass M separated by two springs of negligible mass, each of spring constant k and unstretched length `. The springs are lightly damped to stop vibrations. This is similar to the example shown in **Figure 8**. The lower bar **A** is initially at a height of h above the ground. The ride is then released from rest and falls freely under gravity.

Figure 8: credit: https://www.kerncountyfair.com/p/things-to-do/carnival

(i) Find an expression for the speed of the bars immediately before they hit the ground.

(ii) Bar **A** strikes the ground and is brought to rest almost instantaneously, its kinetic energy all being transferred to heat and sound. Determine the total energy of the system, E, in terms of the subsequent maximum compression of the springs x_m, and gravitational potential energy.

After this, the springs extend again, and then at some moment bar **A** lifts off the ground.

(iii) By considering the forces acting on **A** determine the extension x_e, of the springs at the instant **A** leaves the ground.

(iv) Hence, using energy considerations, deduce an expression for the condition the initial height h must satisfy in order for **A** to leave the ground.

(7)

[25 marks]

Question 4

This question is about electric fields and their application.

a) A thin, isolated, conducting, square metal sheet of area A is given a charge $+Q$.

   (i) State where the charge on the sheet is to be found.

   (ii) Sketch a set of electric field lines (with arrows) emerging from the sheet in the region near the surface and centre of the sheet (i.e. neglect edge effects).

   (iii) How are the field lines qualitatively related to the magnitude of the electric field in a given region?

   (iv) If the field strength near the sheet is E_s, write down the force on a charge $+q$ at a distance d from the sheet, where $d \ll \sqrt{A}$, in terms of E_s and q.

(4)

b) Consider a conducting spherical shell of radius r given a charge $+Q$.

   (i) i. What is the charge density, σ (the charge per unit area), and the electric field strength E at the surface of the sphere?

   ii. Hence write down an expression for the electric field strength E in terms of σ and ε_0 for the conducting, charged surface. How does E depend upon r?

   iii. Hence, for an isolated, thin, plane, conducting surface carrying charge density σ, what is the electric field strength very near the surface?

   (ii) A conducting, charged sphere has no field inside the surface for a charge Q on the surface, but a field E_{sphere} on the outside surface. The repulsion of the surface charges will cause an outward force on the surface, which can be expressed as a pressure P.

   Imagine that this sphere described has a small area ΔA removed, along with its associated charge Δq, determined by the value of σ for the sphere, as illustrated in **Figure 9**. The field lines remain distributed as before since we are not in practice removing the small area.

   i. What is the value of the field strength E_s on each side of the area ΔA, in terms of E_{sphere}?

   ii. Hence write down the field strength in the hole at the spherical surface, just inside where ΔA would sit, due to the charge on the rest of the sphere, in terms of E_{sphere}.

   iii. Hence what is the force on the charge of the area ΔA when it is in place?

   iv. Now derive an expression for the surface pressure on the sphere due to the charge it carries in terms of E_{sphere} and ε_0.

(7)

c) An electric field strength of greater than $3 \times 10^6\,\mathrm{N\,C^{-1}}$ causes air to breakdown and conduct. Determine

   (i) the radius of the smallest water drop (treated as a conducting spherical shell so that any excess charge resides on the surface) that can be charged to an electric potential of 7000 V,

   (ii) the charge, and the surface charge density of the drop.

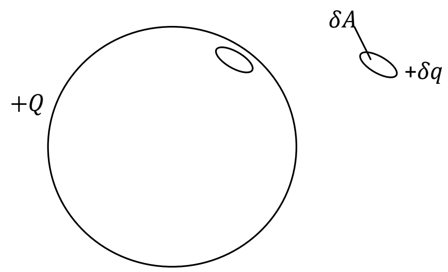


Figure 9

Charged drops of water are produced in a manner similar to that illustrated in **Figure 10** with the capillary tube at $7.0\,\text{kV}$. The drop is released vertically and falls under gravity towards an earthed metal plate $10\,\text{cm}$ below the tube where it was formed.

Calculate

(iii) the speed of the drop when it reaches the earthed metal plate below,

(iv) the average current given a flow rate through the tube of $64\,\text{cm}^3$ per hour.

(7)

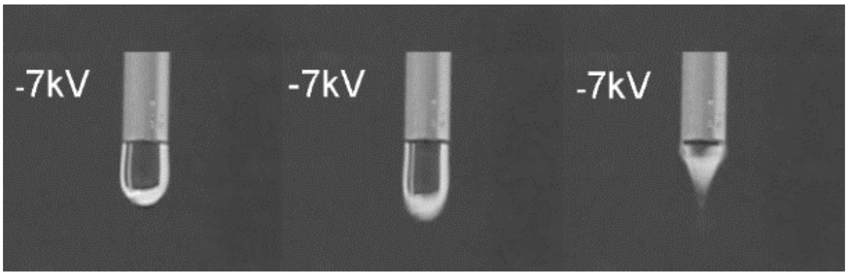


Figure 10: credit: Water with Excess Electric Charge: Leandra P. Santos, Telma R. D. Ducati, Lia B. S. Balestrin, and Fernando Galembeck; J. Phys. Chem. C 2011, 115, 11226–11232

d) The intention is to study the motion of the falling drops by means of high speed photography, and one means of doing this is to use a rotating prism camera shutter. **Figure 11** shows the general arrangement for the film passing through the camera at high speed. The continually moving film would blur the image, so a rotating cube of

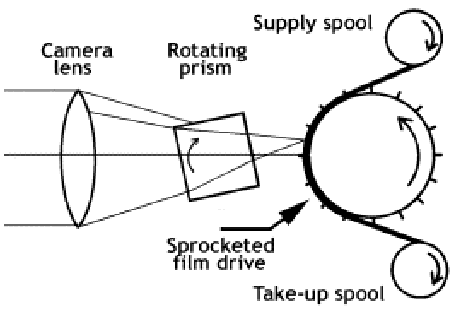


Figure 11: credit: (An Overview of High Speed Photographic Imaging by Andrew Davidhazy, School of Photographic Arts and Sciences, Rochester Institute of Technology)

glass is used to move the image for a short time along the film at the same speed as the film travels through the camera.

Figure 12 shows a ray of light entering the rotating cube of glass of side x, with a refractive index n.

(i) Determine an expression for the lateral displacement d, in terms of $x; n$, and the angle of incidence i, which should be assumed to be small.

(ii) The prism rotates about an axis perpendicular to the plane of the diagram, such that the image is stationary relative to the moving film. How fast should the film move if $x = 5.0\,\text{cm}$, $n = 1.5$ and the cube rotates at $200\,\text{Hz}$? **(7)**

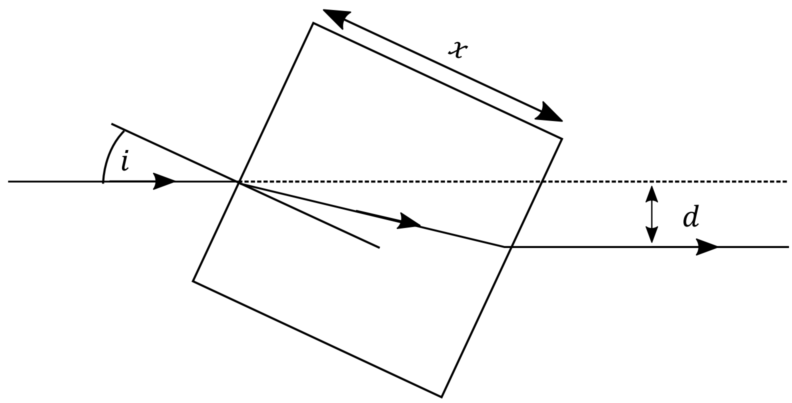


Figure 12

[25 marks]

Question 5

This question is on gravity and forces.

a) In 2017 an object that became known as Oumuamua was observed in the Solar System. **Figure 13** shows an artist's impression.

Figure 13: (Picture: ESO)

The speed of the object was measured to be $87.7\,\mathrm{km\,s^{-1}}$ at a closest approach to the Sun of $38.1 \times 10^6\,\mathrm{km}$ and astronomers deduced that Oumuamua did not belong to the Solar System. Make a suitable calculation to confirm this deduction.

(4)

b) In January 2019 the New Horizons spacecraft made a fly by of a distant object in the Solar System called Ultima Thule, shown in **Figure 14** below. The planetoid consists of two lobes of diameters $2r_1 = 19.5\,\mathrm{km}$ and $2r_2 = 14.2\,\mathrm{km}$. The period of rotation about its short axis is $T = 15\,\mathrm{hours}$. (The long axis is a line passing through the centres of the two lobes, and a short axis is perpendicular to the long axis.)

Figure 14: The two approximately spherical lobes making up Ultima Thule.

Credits: NASA/Johns Hopkins Applied Physics Laboratory/Southwest Research Institute, National Optical Astronomy Observatory

If we assume that the two lobes are held together by a weak gravitational attraction, we can model the object as two spheres of the same density just touching.

(i) Find the centre of mass of Ultima Thule, as measured from the centre of the largest sphere.

(ii) By considering the resultant force acting on one sphere, estimate the density of Ultima Thule.

(iii) Using the fact that the density can be taken to be similar for the two lobes, sketch a graph of the gravitational field strength along the long axis.
Note: the field strength varies linearly with radius inside a spherical body of uniform density.
You may find it helpful to sketch the potential along the long axis.

(8)

In fact, the two lobes are not bound by gravity, so the density and the masses of the lobes are not known. From the appearance of the two lobes, it looks as though they were moving at a low relative speed when they collided.

As an example, consider two spherical iron masses with the same radii as the lobes of Ultima Thule. The masses are $m_1 = 3.06 \times 10^{16}\,\mathrm{kg}$ and $m_2 = 1.18 \times 10^{16}\,\mathrm{kg}$ for the larger and smaller lobes respectively. We can assume that they approached from a great distance apart along a line through their centres, with a negligible initial relative velocity.

(iv) Calculate the amount of energy liberated when these two iron spheres collided.

(v) Calculate the relative speed of approach.

(7)

This energy of collision may have initially led to an increase in temperature of the two spheres (you may neglect any energy of rotation), which we could calculate from knowing the specific heat capacity of iron. However, at the very low temperatures of the outer Solar System the molar heat capacity of a material is not constant. For example, at low temperatures the molar heat capacity of iron, c, varies as

$$c = \frac{12\pi^4 R}{5\Theta_D^3} T^3$$

where:

c is the molar heat capacity in $\mathrm{J\,mol^{-1}\,K^{-1}}$

R is the universal gas constant, $R = 8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$

Θ_D is a constant, $\Theta_D = 464\,\mathrm{K}$ for iron

T is the temperature in kelvin

molar mass of iron $= 56\,\mathrm{g\,mol^{-1}}$

(vi) Calculate the number of moles of iron involved in the collision.

(vii) Using your value of the energy liberated on collision, calculate the maximum temperature reached by the iron spheres, assuming the initial temperature was $4\,\mathrm{K}$ and radiative losses are in the first instance negligible.

(6)

[25 marks]

Question 6

This question explores different applications of wave interference.

a) State the necessary conditions to be able to *observe* stable wave interference patterns.
(2)

b) (i) Consider a ship that carries a radio receiver a height h above the sea, which is anchored (stationary) a large distance D away from the bottom of a cliff. Suppose on top of the cliff there is a radio transmitter at a height of H above the sea and transmitting waves of wavelength . Assuming that the sea acts as a flat, perfectly reflecting surface, show that the signal strength received by the ship is a maximum when

$$h = \frac{D}{4H}:$$

State any assumptions you make. Note that there is a phase change of (180) when waves reflect at an interface going from a less dense to a denser medium.
(4)

(ii) At high tide, the ship is $1:5$ km away from the cliff and its receiver is at 20 m above the sea with the height of the transmitter above the sea at 80 m. The frequency of the radio waves is 70 MHz.

i. By how much would the tide need to fall for the signal received by the ship to become a minimum?

ii. The normal maximum tidal range (twice the amplitude) is 5 m. Would the signal strength change appreciably between low and high tides, and why?

iii. What if the receiver height was just above sea level at $0:5$ m?

(4)

c) A simplified model of sodium chloride (salt) consists of a cubic arrangement of alternating sodium and chloride ions separated by distance d. When a plane wave of X-rays is incident on the ions, each ion can be considered as a new source of spherically scattered new waves.

(i) Show that for plane waves of wavelength and incident at angle to be reflected from the crystal constructively they must satisfy

$$2d \sin \quad = n \; ;$$

where n is a positive integer and is specified as the angle given in **Figure 15**. This is called the *Bragg condition*.
(3)

(ii) The density of salt is $2:17$ g cm ^{3} with a molar mass of $58:4$ g mol ^{1}. Calculate the value of d for salt.
(3)

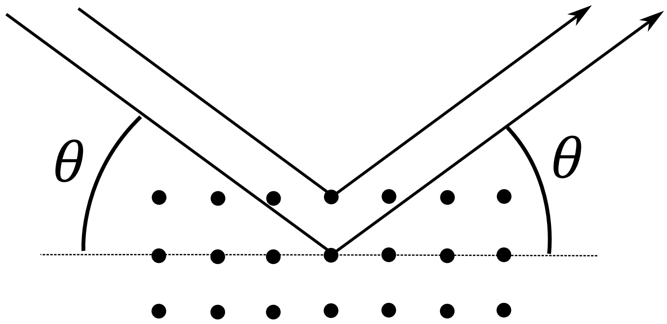


Figure 15: Bragg reflection (an interference condition) with angle θ specified.
The dots denote the atomic layers.

(iii) In an X-ray diffraction experiment, waves that are incident at a minimum angle of 25 are strongly reflected from a salt crystal. Calculate the wavelength of the X-rays.

(2)

d) An oil film on water produces colourful interference effects, as shown in **Figure 16**.

(i) Sketch a diagram showing the path of a ray of light entering and leaving the oil film. Mark on two paths that are going to account for the interference effect observed.

(ii) Calculate the minimum thickness of film needed to observe a strong reflection of violet light with a wavelength of 410 nm, at an angle of reflection of 50 .
The refractive index of oil is 1.5, and that of water is 1.33.

Figure 16: Oil on water.
(image credit: E. Wiebe, http://climate.uvic.ca)

[7]

[25 marks]

END OF SECTION 2

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