British Physics Olympiad 2016-17

Round 2 Competition Paper

Monday 30th January 2017

Instructions

Time: 3 hours (approximately 35 minutes per question).

Questions: All five questions should be attempted.

Marks: The questions carry similar marks.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. **Graph paper should be provided**. Students should ensure their name and school is clearly written on all answer sheets and pages are numbered. A standard formula booklet with standard physical constants should be supplied.

Instructions: To accommodate students sitting the paper at different times, please do not discuss any aspect of the paper on the internet until 8 am Saturday 4th February. **This paper must not be taken out of the exam room**.

Clarity: Solutions must be written legibly, in black pen (the papers are photocopied), and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam paper.

Training Dates and the International Physics Olympiad

Following this round, the best students eligible to represent the UK at the International Physics Olympiad (IPhO) will be invited to attend the **Training Camp** *to be held in the Physics Department at the University of Oxford,* **(Tuesday 4th April to Friday 7th April 2017)**. *Problem solving skills will be developed, practical skills enhanced, as well as some coverage of new material (Thermodynamics, Relativity, etc.). At the Training Camp a practical exam is sat as well as a short Theory Paper. Five students (and a reserve) will be selected for further training. From May there will be mentoring by email to cover some topics and problems. There will be a weekend* **Experimental Training Camp in Oxford 19th – 21st May (Friday evening to Sunday afternoon)**, followed by a **training camp in Cambridge beginning on Thursday 29th June.**

The IPhO this year will be held in Yogyakarta, Indonesia, from 16th to 24th July 2017.

Important Constants

Constant	**Symbol**	**Value**
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Acceleration of free fall at Earth's surface	g	$9.81\,\mathrm{m\,s^{-2}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Mass of a neutron	m_n	$1.67 \times 10^{-27}\,\mathrm{kg}$
Mass of a proton	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$
Boltzmann constant	k	$1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Planck's constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}}$
Mass of the Sun	M_S	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	M_E	$5.97 \times 10^{24}\,\mathrm{kg}$
Radius of the Earth	R_E	$6.38 \times 10^{6}\,\mathrm{m}$
Mass of Mars	M_M	$6.42 \times 10^{23}\,\mathrm{kg}$
Radius of Mars	R_M	$3.40 \times 10^{6}\,\mathrm{m}$

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Qu 1. Estimations and Ideas

Answer the following questions briefly, but including physics ideas and explanations.

(a) A container of water in which a rat is swimming is in equilibrium on a balance (as in a ruler balanced on a centre pivot). A string is lowered from a rod attached to the other side of the balance so that it just touches the water. The rat begins to climb up the string. What happens to the balance?

(b) It is possible to observe a bright star (not the Sun) in the sky in the daytime using an ordinary telescope. Explain why the magnification of the eyepiece is crucial to being able to observe such a star, rather more than the objective lens or mirror of the telescope.

(c) The human eye is used to observe a distant point source of light. If the intensity of the light is I_0 at the eye's pupil, by what factor is the intensity increased on the retina of the eye?

(d) There are six push-button switches. When none of them is pressed, a light bulb remains unlit. When buttons 1, 2 and 5 are simultaneously pressed, the bulb lights. No other combination of buttons can be pressed so as to light the bulb. Draw a diagram of the simplest electric circuit having this behaviour.

(e) The image of the living plant in Figure 1 has ice wings. Suggest how these might have formed.

Figure 1: Plant growing in frozen environment.

(f) Neutrons pass readily through a lead block, but are stopped in the same distance in paraffin, water or other compound containing hydrogen atoms. Why might this be?

(g) An ordinary car is driven along a straight road, but it has been fitted with square wheels instead of the usual round ones. What is an estimate of the extra power converted by the engine in order to drive the car at 30 mph (miles per hour)?

(h) A washer, shown in Figure 2, is made of a conducting material of resistivity, ρ. It has a square cross-section of width a and thickness t, and its outer radius is $2a$. There is a small slit cut in the washer, and wires of negligible resistance are connected to the exposed faces at the slit. What is the resistance of the washer measured between the pair of connecting wires?

(i) A child stores his crayons in a plastic water bottle which has a narrow neck. When he shakes the bottle to get the crayons out, they jam in the neck. However, when several of the crayons have come out, the rest can be shaken out much more easily. Comment on this effect from a physics viewpoint.

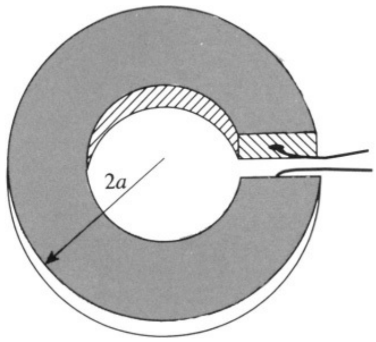

Figure 2: Washer shaped resistor.

Qu 2. Magnetoresistance

Magnetic properties of electrons are intrinsically related to their spin. In ferromagnetic materials, unpaired electron spins line up with each other within regions called domains. Within domains the magnetic field is therefore intense, though domains will generally be randomly orientated with respect to one another so that a bulk sample will usually be unmagnetised. However, an externally applied magnetic field can cause the magnetic domains to line up with each other; the material is said to be magnetised.

Electrons, although point particles, behave as if they were spinning and act like tiny magnets. Electrons in a ferromagnet whose spins are oriented in the direction of (parallel), or opposite to (antiparallel), the internal magnetisation carry independent currents, leading to the material behaving as if it has different resistivities ρ_p and ρ_a for each of the two current components.

Two ferromagnetic layers with identical dimensions (length ℓ, cross-sectional area A) but opposite magnetisation are placed next to each other in parallel with a cell of e.m.f. ε and internal resistance r. The resistance of the rest of the external circuit is modelled by a resistance R in series with the ferromagnetic layers.

(a) Draw a diagram of the circuit described above.

(b) Find the total resistance of the circuit.

(c) An external magnetic field is now applied to the ferromagnetic layers causing their magnetisation to line up. Draw a new circuit diagram and find the total resistance in this case.

(d) The current component parallel flows much more easily than that antiparallel, meaning that $\rho_\text{p} \ll \rho_\text{a}$.

(i) Find an expression for the change in current between the two situations before and after the external field is applied.

(ii) For fixed resistances (and fixed resistivities $\rho_\text{p} \ll \rho_\text{a}$), under what conditions is the change in current maximised?

(e) The effect described above is known as magnetoresistance, and can be employed in the construction of hard disk read/write heads. Suggest how one of these might work and how the result of **(d) (ii)** is relevant. What other considerations are there?

Qu 3. Radioactive Decay

An ionization chamber is designed to measure the ion current from a radioactive gas. Initially the gas in the chamber is nitrogen. A small amount of Gas A is introduced into the chamber. It contains radioactive atoms which emit alpha particles with energy $6.5\,\mathrm{MeV}$. It takes $15.6\,\mathrm{eV}$ to ionize a molecule of nitrogen.

(a) How many alpha particles per second does a current of 17×10^{-9} A correspond to?

(b) Make an estimate of the expected variation in this current due to random variations in radioactive decay. Why do the current readings not show these variations?

(c) For accurate half-life measurements, a balanced ionization chamber apparatus has been used. This is shown schematically in Figure 3. The liquid sample of radium, whose half life is known accurately, and an unknown sample B are inserted in adjacent chambers. The voltage across a $10^{10}\,\Omega$ resistor is measured when it is attached to each chamber to measure the difference in current between them. The measurements go to a recorder. What advantages could there be in this experimental method and setup?

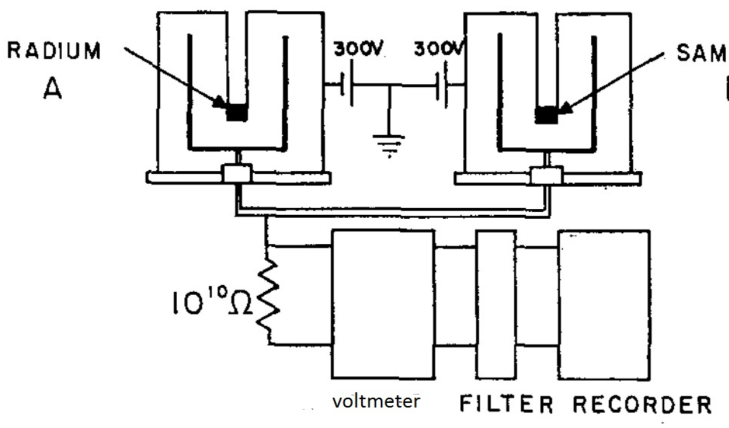

Figure 3: Schematic construction of a balanced ionization chamber apparatus.
credit: Easterday H.T. and R. L. Smith: Nuclear Physics 20 (1960) 155-158

(d) After evacuation of the original ionization chamber, Gas B is introduced and the current measured. Gas B contains two radioactive isotopes, both emitting α radiation of the same energy as Gas A.

Time /min	Current /nA
0	17.000
1	14.930
2	13.261
3	11.912
4	10.820
5	9.934
6	9.212
7	8.622
8	8.137
9	7.737
10	7.404
11	7.126
12	6.892
13	6.692
14	6.521
15	6.372
16	6.242
17	6.126
18	6.022
19	5.927
20	5.841
21	5.760
22	5.685

Plot a graph of $\ln(I/\text{n}A)$ against time (where I is current) and explain its shape in as much detail as you can.

(e) Use your graph to determine the half-life of the longer lived isotope.

(f) Now use the results of these calculations to find the half-life of the shorter lived isotope.

(g) Gas B is removed before Gas C is introduced and the current measured. Gas C is identical to Gas B except that the decay product of the shorter lived isotope is itself unstable, emitting alpha particles of identical energy to Gases A and B. If S is the short-lived component of C, L is the long-lived component and X is the decay product of S, sketch graphs of $\ln(I/\text{n}A)$ against time for the following cases. Indicate rough values on your graphs.

(i) The half-life of X is less than the half-life of S.

(ii) The half-life of X is greater than the half-life of L.

Qu 4. Elliptical Orbits

This question concerns elliptical orbits.

(a) The equation of an ellipse of eccentricity $\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$ and centre (x_0, y_0) in Cartesian co-ordinates is

$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

where a is the semi-major axis (i.e. along its longest diameter) and b is the semi-minor axis (i.e. along its shortest diameter).

Show that the polar equation

$$r = \frac{\alpha}{1 + \varepsilon \cos\theta}$$

describes an ellipse in the x-y plane with the following features:

$$\text{centre } (x_0, y_0) = \left(-\frac{\alpha\varepsilon}{1 - \varepsilon^2}, 0\right)$$

and

$$\text{semi-major axis } a = \frac{\alpha}{1 - \varepsilon^2}$$

and

$$\text{semi-minor axis } b = \frac{\alpha}{\sqrt{1 - \varepsilon^2}}$$

Sketch the ellipse, noting the aforementioned features as well as the locations of the foci which are at a distance $f = \sqrt{a^2 - b^2}$ either side of the centre along the semi-major axis. What constraints are there on the eccentricity? You may find it useful to consider the relationship between cartesian coordinates and polar coordinates: $x = r\cos\theta$, $y = r\sin\theta$, noting that $x^2 + y^2 = r^2$.

(b) A Sun-planet system has total energy E and angular momentum L. M_S is the mass of the Sun and m_p the mass of the planet.

(i) Angular momentum is like a 'moment' of momentum. So, for an object undergoing rotational motion, its angular momentum at a distance r from an axis of rotation with a *linear* speed v (the component perpendicular to r) at that point, will have angular momentum equal to mvr. Furthermore, Newton's second law applies to angular momentum just as it does to momentum if you replace 'resultant force' with 'resultant moment/torque' and 'momentum' with 'angular momentum'.

Why must L be a constant? Show that the rate at which an imaginary line drawn between the orbital axis and the planet sweeps out area is given by

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{L}{2m_p}$$

What relevance does this have for Kepler's second law?

(ii) Show that

$$E = \frac{1}{2}m_p\dot{r}^2 + \frac{L^2}{2m_p r^2} - \frac{k}{r}$$

where $\dot{r} = \left(\frac{\mathrm{d}r}{\mathrm{d}t}\right)$ and k is a constant to be determined.

If time, t, is re-parametrised in terms of angle, θ, the solution to this differential equation is $r = \alpha/(1 + \varepsilon\cos\theta)$ with $\alpha = L^2/\mu k$ and $\varepsilon^2 = 1 + \frac{2EL^2}{\mu k^2}$.

(iii) Find a formula for the time period of the planet in terms of the semi-major axis of its orbit. The total area of an ellipse is πab.

(c) A satellite in low earth orbit at an altitude of $320\,\mathrm{km}$ is to be transferred to a geosynchronous orbit using an elliptical orbit which is tangential to both the initial and final orbits at its periapsis and apoapsis (the smallest and largest distances from a focus) respectively. How elliptical is the transfer orbit, what is the total velocity change, Δv, required to achieve the transfer and how long will it take? State any assumptions you make.

(d) A significant part of a manned mission to Mars involves transfer from a low earth orbit of altitude about $300\,\mathrm{km}$ to a low orbit around Mars of altitude about $250\,\mathrm{km}$. What is the Δv budget needed to make the trip using a similar transfer to that in **(c)**, and what is the shortest length of time for a return trip using this method? What are your assumptions and how good are they in this context? To three significant figures, the eccentricites of Earth and Mars are 0.0167 and 0.0934 respectively, while Mars takes 1.88 Earth years to orbit the Sun.

Qu 5. Thermal Properties of Ideal Gases

The first law of thermodynamics can be written as

$$\delta U = \delta Q + \delta W$$

where δU is the change in internal energy of a given system, δQ is the heat supplied to the system and $\delta W = -p\delta V$ is the work done on the system.

(a) Show that for a monatomic ideal gas, the heat capacity at constant volume is given by $C_V = \frac{3}{2}nR$, and also that the heat capacity at constant pressure is given by $C_p = C_V + nR = \frac{5}{2}nR$. n is the number of moles of gas.

(b) When a monatomic ideal gas undergoes an adiabatic change, no heat is supplied to the gas leading to $pV^\gamma =$ const., where $\gamma = \frac{C_p}{C_V}$. Determine the corresponding relationships between V and T, and between p and T.

(c) Two thermally isolated (i.e. insulated) cylinders, L and R, of equal volume, both equipped with pistons, are connected by a valve. Initially L has its piston fully withdrawn and contains an ideal monatomic gas at temperature T_0, while R has its piston fully inserted and the valve is closed as shown in the diagram below:

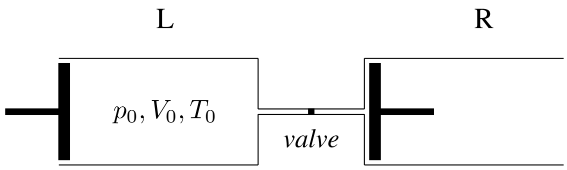

Figure 4: Thermally isolated cylinders connected by a valve which can be opened and closed.

Calculate the final temperature of the gas in each cylinder after the following operations are performed, each with the same initial configuration as already described. Assume that the thermal capacity of the cylinders is negligible.

(i) The valve is fully opened and the gas is slowly drawn into R by pulling out piston R; piston L remains stationary.

(ii) Piston R is fully withdrawn and the valve opened slightly. The gas is then driven as far as it will go by pushing in piston L at such a rate that the pressure in L remains constant. The cylinders are in thermal contact.

(iii) Piston R is fully withdrawn and the valve opened slightly. The gas is then driven as far as it will go by pushing in piston L at such a rate that the pressure in L remains constant. The cylinders are thermally isolated from each other.

(iv) Piston R is fully withdrawn and the valve is then opened. The cylinders are in thermal contact.

(v) Piston R is fully withdrawn and the valve is then opened. The cylinders are thermally isolated from each other.

END OF PAPER

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