BAAO
British Astronomy and Astrophysics Olympiad

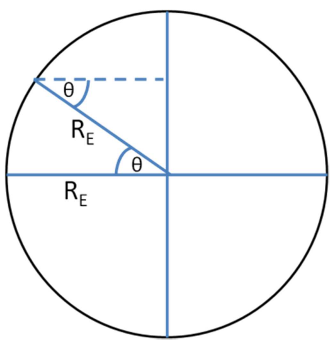
Astronomy & Astrophysics A2 Challenge

September - December 2016

Solutions and marking guidelines

- The total mark for each question is in **bold** on the right hand side of the table. The breakdown of the mark is below it.
- There is an explanation for each correct answer for the multiple-choice questions. However, the students are only required to write the letter corresponding to the right answer.
- In Section C, students should attempt **either** Qu 13 **or** Qu 14. If both are attempted, consider the question with the higher mark.
- Answers to one or two significant figures are generally acceptable. The solution may give more in order to make the calculation clear.
- There are multiple ways to solve some of the questions, please accept all good solutions that arrive at the correct answer.

Question	Answer	Mark
Section A		**10**
1.	D The Coriolis force is an inertial force that acts on objects that are in motion relative to a rotating reference frame. As a result, hurricanes rotate anti-clockwise in the northern hemisphere and clockwise in the southern hemisphere. (this is also explained through cons. of angular mom.)	1
2.	B There are two tides every day: one caused by the gravitational attraction of the Moon on the side facing it, and the other due to the centrifugal force experienced by the side furthest from the centre of mass.	1
3.	D The number of photons collected is proportional to the area of the aperture, which is proportional to the diameter^2. Therefore if the diameter is 4 times bigger it will receive $4^2 = 16$ times more photons.	1
4.	A Aquila is not a zodiacal constellation. According to the astronomical definition, there are 13 zodiacal constellations, with Ophiucus being the least known of them.	1
5.	D The orbit of the Moon is inclined 5° to the ecliptic. During winter, the Sun has its lowest declination. Thus the Full Moon, which is on the opposite side of the sky, will have its highest declination and will be visible highest in the sky.	1
6.	B Each fold doubles it, so new thickness after n folds is $10\ \mu m \times 2^n$. Setting this equal to 1 AU gives $n = 53.7$ so closest is 50.	1

7.	C The altitude of Polaris (Northern Star) above the horizon is the latitude of the observer. Thus, at a latitude of 52° N, the altitude of Polaris is 52°.	1
8.	A The rising and setting sun alignments will happen an equal number of days before and after the winter and summer solstices respectively (roughly 21st June and 21st December). Since 11th July is 21 days after the summer solstice, you were looking for a date about 21 days before (in order to get a setting Sun).	1
9.	C The first clue is that you are not at the North Pole, which would be a trivial solution. By looking at the answers, the second clue is that it has to be close to one of the Poles (South Pole more specifically, as the first direction of travel is South). You start 100 Miles North of a line of latitude whose entire length is 100 Miles. You travel South 100 Miles, then East 100 Miles until you encircle the South Pole completely, then travel back North 100 Miles to finish where you started. The radius of a circle with the circumference of 100 Miles is $\frac{100}{2\pi} = 16$ **Miles.** Thus, you have to start 116 Miles from the South Pole. This is only an approximate answer, as it assumes the Earth to be flat. However, the Earth does not curve much on the small distances we considered in the question, a full treatment on a sphere would give an answer just 7 cm away.	1
10.	B The angular size of a pixel is $\frac{2.2°}{2024} = 0.0011 \frac{°}{\text{px}} = 1.9 \times 10^{-5} \frac{\text{rad}}{\text{px}}$ The distance the image was taken at is: $\frac{1\text{ m}}{25\text{ px} \times 1.9 \times 10^{-5}\frac{\text{rad}}{\text{px}}} = 2.1\text{ km}$	1
Section B		**10**
11.	a. i. **Answer:** 463 m s^{-1} Sun will appear stationary if the Eurofighter moves at the Earth's rotational speed, so: $$v_{equator} = \frac{2\pi R_E}{T} = \frac{2\pi \times 6.37 \times 10^6}{24 \times 60 \times 60} = 463\text{ m s}^{-1}$$ ii. **Answer:** 285 m s^{-1} Oxford is less far from the Earth's rotational axis, so the speed needed will be less by a factor of cos θ (where θ is the latitude of Oxford) $$v_{oxford} = \frac{2\pi \times R_E \cos\theta}{T} = 285\text{ m s}^{-1}$$  b.	**2** 1 1 **3**

	At the Equator a day is 12 hours (due to the value of $v_{equator}$), and during a day the Sun travels 180°, so since it is 0.5° across it covers that angular distance in: $\frac{0.5}{180} \times 12 = 0.033 \text{ hours} = 2 \text{ minutes}$ In the Eurofighter the effective Earth rotation speed becomes: $500 - 463 = 37\ m\ s^{-1} = 0.08\ v_{equator}$ So time to rise the sun by 0.5° becomes: $\frac{2 \text{ minutes}}{0.08} = 25 \text{ minutes}$	1 1 1
12.	a. **Answer:** 12 900 km Need ratio of diameter (or radius) and distance to be the same, so: $\frac{2R_\odot}{1\text{ AU}} = \frac{120\text{ km}}{d}$ d = 12 900 km This is a much higher altitude than the ISS. [Note 1: if the student forgets to convert both into radii or into diameters (i.e. gets d = 25 800 km) then they lose the first mark, but can get 2 ecf marks] [Note 2: students can use the comparison with angular diameter of 0.5° from the previous question for the first mark, but must recognise that they need to convert the angle into radians to get the second mark (if done correctly they'll get d = 13 800 km)] b. **Answer**: 7.41 hours Using Kepler's Third Law: $T^2 = \frac{4\pi^2}{GM}a^3$ $\therefore T = \sqrt{\frac{4\pi^2}{GM_E}(R_E + d)^3} = \sqrt{\frac{4\pi^2}{6.67\times10^{-11}\times5.97\times10^{24}}(6.37\times10^6 + 12.9\times10^6)^3}$ T = 2.67×10^4 s (= 7.41 hours) [Note 1: first mark is for recognition that the radius of the orbit is equal to the radius of the Earth plus the altitude, expressed either algebraically or through the substitution] [Note 2: allow ecf from part a. so long as value for final period is sensible]	**3** 1 1 1 **2** 1 1

Section C	**Either** Qu 13 **or** Q 14	**10**
13.	a.	**4**
	At Roche limit $F_{grav} = F_{tidal}$, so: $$\frac{Gmu}{r^2} = \frac{2GMur}{d_{RL}^3}$$	
	Correct rearrangement and cancelling: $$d_{RL} = r\left(2\frac{M}{m}\right)^{\frac{1}{3}}$$	1
	Mass of planet and satellite in terms of density: But $M = \frac{4}{3}\pi\rho_M R^3$ and $m = \frac{4}{3}\pi\rho_m r^3$	1
	Substitution of densities into equation: $$\therefore d_{RL} = r\left(2\frac{\rho_M R^3}{\rho_m r^3}\right)^{\frac{1}{3}}$$	1
	Cancelling of r to leave required expression: $$\therefore d_{RL} = R\left(2\frac{\rho_M}{\rho_m}\right)^{1/3}$$	1
	b.	**2**
	$$\rho_{Saturn} = \frac{M_{Saturn}}{\frac{4}{3}\pi R_{Saturn}^3} = \frac{5.68\times10^{26}}{\frac{4}{3}\pi\times(6.027\times10^7)^3} = 619 \text{ kg m}^{-3}$$	1
	$$d_{RL} = R_{Saturn}\left(2\frac{\rho_{Saturn}}{\rho_{ice}}\right)^{1/3} = 60\,270\left(2\frac{619}{930}\right)^{1/3} = 66\,300 \text{ km}$$ $(= 1.10\ R_{Saturn})$	1
	c.	**1**
	$$d_{RL,max} = 2.44\times 60\,270\left(\frac{619}{930}\right)^{1/3} = 128\,400 \text{ km } (= 2.13\ R_{Saturn})$$	1
	d.	**1**
	The limits (roughly) agree with the observed extent of the ring system [Allow students to say the inner edge agrees well with the simple model, but the outer edge is a poorer fit]	1
	e.	**2**
	Finding the density of a moon that reached the fluid Roche limit at 2 R_{Saturn}: $$\rho_m = \frac{\rho_M}{(2/2.44)^3} = \frac{619}{(2/2.44)^3} = 1124 \text{ kg m}^{-3}$$	1

	Assuming Veritas was spherical then the radius is: $r = \left(\frac{M_{ring}}{\frac{4}{3}\pi\rho_m}\right)^{1/3} = \left(\frac{3\times10^{19}}{\frac{4}{3}\pi\times1124}\right)^{1/3} = 1.85\times10^5$ m (= 185 km) [Given that this is similar in size to several other moons of Saturn, the idea that the rings came from the tidal destruction of a moon is not completely outlandish]	1
14	a. The period is the time interval between two consecutive peaks of the blue curve. From the radial velocity curve, the period is 11 days. [Full marks for the period within ± 2 days]	**1** 1
	b. Using Kepler's Third Law: $$\frac{T^2}{a^3} = \frac{4\pi^2}{GM}$$ The semi-major axis of the planet's orbit is: $$a = \left(\frac{GM\,T^2}{4\pi^2}\right)^{1/3} = 7.16\times10^9\ \mathrm{m} = 0.048\ \mathrm{AU}$$ [Note: allow ecf from part a. so long as value for the semi-major axis is sensible]	**1** 1
	c. The orbital velocity for a circular orbit is: $$v_{circ} = \frac{2\pi a}{T}$$ $$v_{circ} = 47.30\ \mathrm{km\ s^{-1}}$$ [Note: allow ecf from part a. so long as value is sensible]	**1** 1
	d. The total linear momentum in the centre of mass frame is: $\vec{p}_{CM} = 0$ $$\therefore\ \vec{p}_{star} + \vec{p}_{planet} = 0$$ $$\therefore Mv_{star} - mv_{planet} = 0$$ $$\therefore m = M\frac{v_{star}}{v_{planet}}$$ Using $v_{star} = 5\ \mathrm{km\ hour^{-1}} = 0.0014\ \mathrm{km\ s^{-1}}$ and $v_{planet} = 47.30\ \mathrm{km\ s^{-1}}$: $$m = 0.12\times2\times10^{30}\times\frac{0.0014}{47.3} = 7.1\times10^{24}\ \mathrm{kg} = 1.19\ M_{Earth}$$ This is a lower estimate for the mass of the planet because the inclination of	**3** 1 1

	the planet's orbit is not known. Astronomers are only able to measure the radial velocity of the star, not the tangential one. In the calculations above we assumed that the plane of the orbit is in the line of sight, thus the radial velocities are the maximum, total velocities. Therefore, the mass we determined is a minimum mass for the planet; it is in fact $m \sin i$, where i is the inclination of the orbit.	1
	e. We are given that the orbit is an ellipse, hence using the diagram on page 2, we can determine the minimum and maximum distance from the planet to the star. The minimum distance (also known as periapsis) is: $$r_{min} = a(1-e)$$ $$r_{min} = 0.048(1-0.35) = 0.031 \text{ AU}$$ The maximum distance (also known as apoapsis) is: $$r_{max} = a(1+e)$$ $$r_{min} = 0.048(1+0.35) = 0.065 \text{ AU}$$ Thus, the distance from the planet to Proxima Centauri ranges between 0.031AU and 0.065 AU. Hence the maximum equilibrium temperature of the planet is: $$T_{Planet,max} = T_{Star}\left(\frac{R_{Star}}{2r_{min}}\right)^{1/2}$$ $$T_{Planet,max} = 307 \text{ K}$$ The minimum equilibrium temperature of the planet is: $$T_{Planet,min} = T_{Star}\left(\frac{R_{Star}}{2r_{max}}\right)^{1/2}$$ $$T_{Planet,min} = 212 \text{ K}$$ The temperature of the planet ranges between 212 K and 307 K, or -61°C and 34°C. The habitable zone is the band around a star where a planet can have water on its surface in liquid form, at normal pressure. Hence, the equilibrium temperature of the planet must be between 0°C and 100°C. The temperature on Proxima Centauri B reaches values above 0°C, thus it is in the habitable zone of its host star.	**3** 1 1 1
	f. The distance to Proxima Centauri, the nearest star to the Sun, is 4.22 light years. Assuming that the lifetime of a human being is 80 years, the robotic space probe would need to travel at a speed of 15 800 km s^{-1} to reach the star in a lifetime. Such high velocities have not been achieved yet, currently the fastest man-made object is the Juno Mission, travelling at 40 km s^{-1}. Thus, the prospects of reaching Proxima Centauri B in our lifetime are low, unless new technology is developed.	**1** 1