Earth questions solutions

What is the mass of the Earth's atmosphere?

Atmospheric pressure is about 1 x 10^5 Pa (1 Pa is 1 N m^{-2}). So on a square metre we have 10^5 N, and since g = 10 N kg^{-1}, a square metre has a column of air which is 10,000 kg (or 10 tonnes) sitting on it.

The radius of the earth is 6,400 km and the surface area of a sphere is $4\pi r^2$, resulting in the total mass of air being given by $10^4 \times 4 \times \pi \times (6.4 \times 10^6)^2 \approx 5 \times 10^{18}$ kg or 5,000 million million tonnes.

Is g the same value higher up in the atmosphere? Over the height of the atmosphere, say 200 km, g does not decrease by more than 2-3% as this is so small compared to the radius of the earth. If you draw the earth and then the height of the atmosphere, you will realise that the atmosphere forms a very thin coating.

If we thought that we might calculate the height of the atmosphere then we start to see that this is not so simple.

The pressure at the bottom of a column of material of height h and density ρ is given by $P = \rho g h$. Using $P = 10^5$ Pa and ρ = 1 kg m^{-3} for air, $h \approx 10$ km high which is quite unrealistic. Here we have assumed that the density of air, ρ is constant, but due to air being every compressible it is much denser at ground level than a few kilometres up. So our calculation fails to give a sensible result here (Mt Everest is about 8 km high).

How thick would the atmosphere be in this image?

On this diagram of the earth the radius is about 5.7 cm. The height of the atmosphere is not one definite value, but depends on whether you want to have a satellite in a low but short-lived orbit (about 200 km) or a higher longer living orbit (400 km) as there are relatively few molecules to exert drag on the satellite.

This means that on this picture of the earth the atmosphere would be between 1.7 mm and 3.4 mm thick (the thickness of a very fat pencil line). This means that g, the gravitational field strength (the acceleration due to gravity) is going to be almost the same value at the top of the atmosphere as at the bottom.