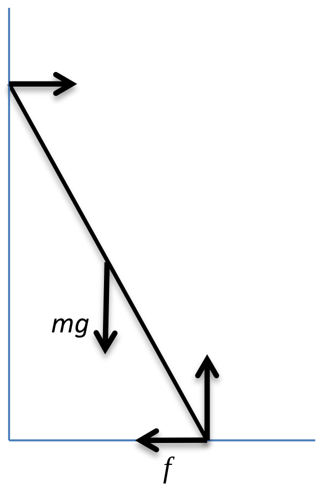
BPhO Leaflet answers

1. How is it that you can lean a ladder…..

    - Draw a diagram of the situation.
    - Remember that if the ladder is to remain there and not to fall (accelerate) away, then the forces must be in equilibrium, and that includes the turning forces i.e. there is no resultant force and no resultant torque, to use the technical terms.
    - Frictional forces (*f*) oppose the motion of the object which would otherwise slide over the surface (the ends of the ladder in this case). There is no frictional force on a smooth surface.
    - The surface also provides a normal reaction force (we could combine the frictional force with the reaction force to give a resultant, but it is clearer if they are resolved, as in the diagram)

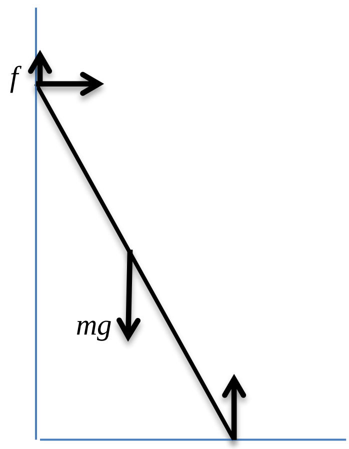
    On the rough ground, the forces vertically and the forces horizontally can cancel out so that there is no resultant force.

    On the smooth ground, there is a resultant horizontal force which will cause the ladder not to be in equilibrium. It may look as though the top of the ladder is being pushed away from the wall rather than the experience of the foot of the ladder sliding away. We show the ladder in a position which it would not expect to be in as it is unstable. The fact that it has a resultant force merely tells you that it will not remain there. If it moved away from the wall with the force shown, then that force would disappear



Rough ground

Smooth wall



Smooth ground

Rough wall

2. A cubic metre of air has a mass of about a kilogram (it is about 1/1000th of the density of water). Sound is transmitted through air by the movement of the molecules subject to a slight change of pressure in the gas. By and large, the speed of sound is similar to the molecules speed, although the molecules do not move along the direction of the sound and so they move a little faster than the speed of sound in the air. The speed of sound is 330 m s^{-1} and the molecules move at about 500 m s^{-1}

(i.e. about 1,000 mph). We do not need to know how many molecules there are as the average speed applies to them all and we know the total mass. Calculation of $\frac{1}{2}mv^2$ gives 125 kJ, which is similar to energy of a 500 kg car moving at about 50 mph (25 m s^{-1}) or 156 kJ.
Air contains a lot of energy!

3. The sun radiates heat and light so that about 1.3 kW m^{-2} arrives at the earth. You could run a washing machine from the power that you could collect in a large umbrella (in principle). The earth is 150 million km from the sun so that if we draw a spherical surface around the sun with this radius, every square metre will receive 1.3 kW.
The energy radiated by the sun every second will therefore be given by $4\pi r^2$ x 1.3 kJ, which is equal to 3.7 x 10^{26} J.
If we use Einstein's famous equation pointing out the equivalence of mass and energy, $E = mc^2$, with c equal to the speed of light (3.0 x 10^8 m s^{-1}), we obtain a mass per second loss of 4.1 x 10^9 kg. This is about 4 million tonnes per second, not of matter blown off but simply a mass loss due to electromagnetic radiation carrying energy out into space.
Can the sun continue to suffer such a loss? Yes, at 2 x 10^{30} kg it is good for another 6 billion (10^9) years.

4. A horse weighs…..

Why does the horse need to eat? Ans: in order to stay warm. Why does the size matter? Because the larger the surface area the more heat (energy per second) is lost. However, the mouse doesn't have much body mass to generate heat and although it does indeed have a much smaller surface area than the horse, **relative to the masses** of the two animals, the mouse really has more surface area.

Consider the food intake/day i.e. power in:
Power generated α food intake/day (each cell requires energy and metabolises at the same rate)

Power lost α surface area of animal

At equilibrium power lost = power generated

So food intake/day α surface area of animal
Surface area α L^2 where L is a linear size of the animal
and Mass α Volume α L^3
So food intake/day α L^2 α $M^{2/3}$

$$\frac{food\ intake\ of\ mouse/day}{food\ intake\ of\ horse/day} = \left(\frac{M_{mouse}}{M_{horse}}\right)^{2/3}$$

Food intake of mouse/day = $80 \times \left(\frac{1}{10^5}\right)^{2/3}$ = 0.037 kg/day
= 37 g/day

i.e. the mouse needs to eat its own bodyweight in 4.6 days ≈ 5 days

5. How many carbon atoms……

   Full stop is, say, 0.5 mm x 0.5 mm. Its thickness is somewhat greater than the wavelength of light as that does not penetrate through to the paper e.g 10 λ i.e. 5,000 nm = 5/1000th mm = 5×10^{-6} m
   Volume of full stop is 1.25×10^{-12} m^3
   Density of graphite is about 3 g cm^{-3} = 3,000 kg m^{-3}
   Mass of graphite is 3.75×10^{-9} kg ≈ 4×10^{-9} kg = 4×10^{-6} g
   Molar mass is 12 g mol^{-1}, so we have $4 \times 10^{-6}/12$ moles = 3×10^{-7} moles
   Avogadro's no. is 6×10^{23} mol^{-1} resulting in 180×10^{20} = 2×10^{15} atoms in the full stop.
   An estimate is therefore 10^{15} carbon atoms.

6. How does a nut and bolt…..

   A question answered at various levels. You may wonder what is not obvious how it works. So why does a nut and bolt not unscrew? A stress applied to the metal plates causes a deformation in the plates and so the frictional force on the nut is greatly increased. In this manner it will not readily unscrew.

7. What is the drift speed…..

   The average drift speed is zero as it is AC. However this is rather an uninteresting result and so we should look a little further. The current *I* in a wire is given by $I = nAve$ where *n* is the charge per unit volume, v is the drift speed of the electrons along the wire, *A* is the cross sectional area of the wire and *e* is the magnitude of the charge on an electron. (This result is simply derived by sketching a length of wire and considering the rate of flow of charge past a point and setting that equal to the current flow).
   A kettle takes about 10 A, $n \approx 10^{29}$ m^{-3} for copper (use Avogadro's No, density of copper ≈ 6 g cm^{-3}, mass number for copper ≈ 64 g mol^{-1}), $A \approx 1 - 2$ mm^2 , $e = 1.6 \times 10^{-19}$ C
   This gives $v \approx \dfrac{10}{10^{29} \text{x } 2\text{x}10^{-6} \text{ x } 1.6 \text{ x } 10^{-19}} \approx$ ⅓ mm s^{-1}
   Since 10 A is the RMS current, the peak current should be greater by $\sqrt{2}$, This is within the accuracy of our estimate but we could include it to give $v_{max} \approx$ ½ mm s^{-1}.
   One might ask how far the electrons travel whilst the kettle is switched on. They travel sinusoidally with the AC so that they keep traveling back and forth. The maths is just that of an oscillating system, for which $v_{max} = A\omega$
   A is the amplitude of the motion and ω is the angular frequency ($=2\pi f$). For 50 Hz, A ≈ 1/600th mm

8. In the summer, why does leaving open the door of the fridge…..

   The thermal energy (internal energy of the gas to be more correct) inside the fridge has to be moved outside into the room. This is a quantity which has to be accounted for just like money. Work is done in order to move this energy by a mechanical process driven by an electric motor behind the fridge. This itself does work or consumes electrical power converting it into heat. So, more heat is produced when you move the energy from inside the fridge to outside, and the result is that the room gets warmer.

9. Estimate the mass of the earth's atmosphere.

Atmospheric pressure is about 1×10^5 Pa (1 Pa is 1 N m^{-2}). So on a square metre we have 10^5 N, and since g = 10 N kg^{-1}, a square metre has a column of air which is 10,000 kg (or 10 tonnes) sitting on it. The radius of the earth is 6,400 km and the surface area of a sphere is $4\pi r^2$, resulting in the total mass of air being given by $10^4 \times 4 \times \pi \times (6.4 \times 10^6)^2 \approx 5 \times 10^{18}$ kg or 5,000 million million tonnes.
Is g the same value higher up in the atmosphere? Over the height of the atmosphere, say 200 km, g does not decrease by more than 2-3% as this is so small compared to the radius of the earth. If you draw the earth and then the height of the atmosphere, you will realise that the atmosphere forms a very thin coating.

If we thought that we might calculate the height of the atmosphere then we start to see that this is not so simple.

The pressure at he bottom of a column of material of height h and density ρ is given by $P = \rho g h$. Using $P = 10^5$ Pa and ρ = 1 kg m^{-3} for air, $h \approx 10$ km high which is quite unrealistic. Here we have assumed that the density of air, ρ is constant, but due to air being every compressible it is much denser at ground level than a few kilometres up. So our calculation fails to give a sensible result here (Mt Everest is about 8 km high).