

A2-2007 Q4

A piece of elastic string is laid around the circumference of a sphere of radius,  $r$ . It is then stretched so that it is raised 1m above the surface. How much longer must the string need to be? How does this extra length relate to the radius,  $r$ , of the sphere?

How much greater would the circumference of the earth become if it was to expand slightly so that its radius increased by 0.1m ?

If in expanding, the radius of the earth increased by  $2 \times 10^{-5} \%$ , then what is the percentage increase in the surface area?

(7 marks)

## A2-2012 Q4

The Physical Review is a distinguished physics journal that has been published continuously since 1893. We will assume that a volume is published twice a year. After about 1935 there was a sharp increase in the number of articles that were published in each volume making it thicker and thicker such that, when it was stacked on the library shelf every six months, the front cover of the journal could be said to be moving along with an ever increasing velocity. A physicist at the time pointed out that if this continued then the front cover of the journal would eventually exceed the speed of light (but then according to relativity theory there would be no information transmitted).

We assume a simple model:

- that the number of articles per volume increase exponentially, doubling every six months (the articles are of similar length)
- at the beginning of 1935 the volume was 1 cm thick
- that the velocity of the front cover is the thickness of that volume divided by the number of seconds in six months.

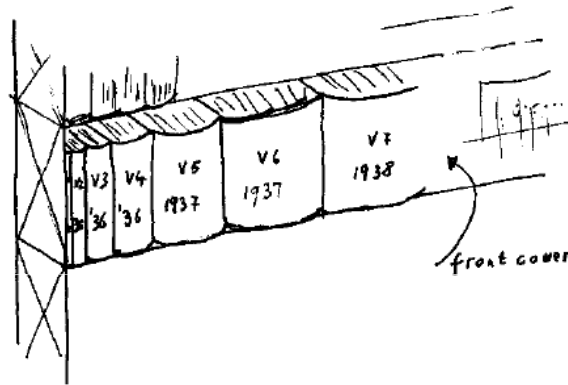


Figure 2. Volumes of the Physical Review doubling in thickness every six months.

You must show your working in the following questions.

- By what factor does the thickness of the volume increase each year?
- What would be the thickness of the volume put on the shelf at the beginning of 1940?
- Write down your answer to (b) to one significant figure and in standard form.
- Using your answer to part (c), write down the thickness of each volume for the next three years.
- Determine the year when the front cover of the volume stacked will exceed the velocity of light. You may find it helpful to write your answer to part (d) including a term of the form  $4^n$ .

Hint: use logs to base 10

$$\log_{10} x^n = n \log_{10} x$$

(10 marks)

## A2-2010 Q2

An insight into the solution of a problem can often be made by looking at the dimensions of the relevant physical quantities.

An example is the simple pendulum, in which a mass at the end of a light inextensible string swings from side to side. The period of the swing,  $T$ , could be determined by resolving the forces acting on the mass. Alternatively, if we suggest that the relevant factors affecting the period are the length of the string,  $l$ , the mass,  $m$ , and the strength of the gravitational field,  $g$ , then  $T$  must depend upon the product of powers of the quantities  $l$ ,  $m$  and  $g$ .

$$\text{i.e. } T = \text{const} \times l^a \times m^b \times g^c \quad (1)$$

The dimensions of  $l$ ,  $m$ ,  $g$  are given by

$$[l] = L, \quad [m] = M, \quad [g] = LT^{-2} \text{ and } [T] = T. \quad (2)$$

So then we can write the equation in terms of dimensions as

$$T = L^a \times M^b \times (LT^{-2})^c \quad (3)$$

The powers of  $T$ ,  $L$ ,  $M$  on each side of the equation must be the same.

For  $T$ :  $T^1 = T^{-2c}$  so that  $c = -1/2$ .

For  $M$ :  $M^0 = M^b$  so  $b = 0$ .

For  $L$ :  $L^0 = L^{a+c}$ , so that  $a = -1/2$ .

This results in the equation  $T = \text{const} \times \sqrt{\frac{l}{g}}$ . A full analysis of the forces will enable you to deduce that  $\text{const} = 2\pi$ .

Now solve the following example in the same manner: when a river floods, large boulders can be left behind on the riverbed, and yet the speed of the river does not change very much (the slope remains the same). Assume that the mass of boulders swept along by the river,  $m$ , depends upon the speed of the river,  $v$ , the gravitational field strength,  $g$ , and the density of the boulder,  $\rho$ .

Write down:

- i) The form of the equation relating  $m$  to  $v$ ,  $g$ ,  $\rho$  as exemplified in equation (1) (1 mark)
- ii) The dimensions of each of the quantities  $m$ ,  $v$ ,  $g$ ,  $\rho$ , as in the set of equations in (2). (4 marks)

## A2-2010 Q2 (continued)

- iii) The dimensional equation for these quantities, as in (3) and solve to obtain the powers  $a$ ,  $b$ ,  $c$ .  
(4 marks)
- iv) The final equation for  $m$  in terms of the variables.  
(2 marks)
- v) From your solution, explain why a flooding river which has a small change of speed has a significant effect on the size of the boulders that are swept along.  
(1 mark)

**[Q2: 12 marks]**